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The Association
of
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THE PRACTICAL STEEL COLUMN.

By GEORGE D. MOUAT, B.Sc.

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PREFACE.

In view of the revisions to British Standard Specification No. 449 (1948) the present is an opportune time to publish a pamphlet on the design of columns.

The earliest investigation of which there is any record was by Musschenbroek in 1729, followed by Euler's fundamental work in 1757. Since then a very extensive literature deals with the theoretical and experimental side but information on the practical design of columns is scanty and writers appear to ignore this aspect of the subject. It is hoped that this work will do something towards rectifying this omission and aid the designer in correlating the latest recommendations to practice.

George D. Mouat.

SYNOPSIS.

Introduction—Euler's crippling stress and loading for standard and other cases—Indication of maximum economy—Gordon-Rankine formula—Ayrton-Perry formula—Load factor, B.S.S. 449 —Permissible stresses for axial loading—Effective length factors—Method of calculating column strengths when axially loaded—End restraint—Intentional eccentricity—Point of application of load—L.C.C. Code—Permissible stresses for axial and non-axial loading—B.S.S. 449 formula for combined compressive and bending stresses—Comparison of strengths of columns of equal area—Distribution of moments and application to columns—Load factor diagram—Examples of calculations for single, double and treble length columns—Wind loading—Appendix—Basic theory for calculating equivalent eccentricities.

THE PRACTICAL STEEL COLUMN.

George D. Mouat, B.Sc.

INTRODUCTION.

By the term column is understood the continuing vertical members supporting the floors in a building or other structure. Pillars, posts, stanchions, struts, etc., while generally similar regarding their use only support one floor and as is to be expected, the continuing column is developed from that which has a single length. The term strut will be used throughout the text for this type.

It is now accepted that, in practice, it is impossible to obtain a perfectly straight column. The numerous causes for this lack of straightness do not, however, occur simultaneously and perfection in columns is unattainable due to working up of material in fabrication. L. Tetmajer, who was an important experimenter, found that the modulus of elasticity varies even in the same piece of material and J. Christie writing in the Transactions of the American Society of Civil Engineers concludes that the elastic axis and the geometric axis do not coincide. Columns which are vertical when erected often become out of plumb. This may be due to uneven settlement of foundations which would produce curvature if nothing else and curvature must also be caused by the moments which are transmitted by the ends of beams framing into columns. It follows that no column can have truly concentric loading although the actual eccentricity may be small.

Euler's Crippling Stress.—Euler, using the fundamental formula IE $d^2y/dx^2 =$ bending moment and assuming a long slender strut originally straight to be deflected by a lateral force deduced a crippling stress. At some value of the load P, in Figure 1, the strut, which must necessarily be deflected by the lateral force will not become straight on the removal of the lateral force. In this state any addition to P causes increased deflection, hence increased bending stress and ultimate collapse of the strut. The lowest value of P at which the strut does not become straight on the removal of the lateral force is the critical or ultimate load for the strut and is the load at which bending just commences to occur.

The standard case to which all others are referred is that of round ends, Figure 1, and since Euler's crippling stress is basic, it is again recorded here :—

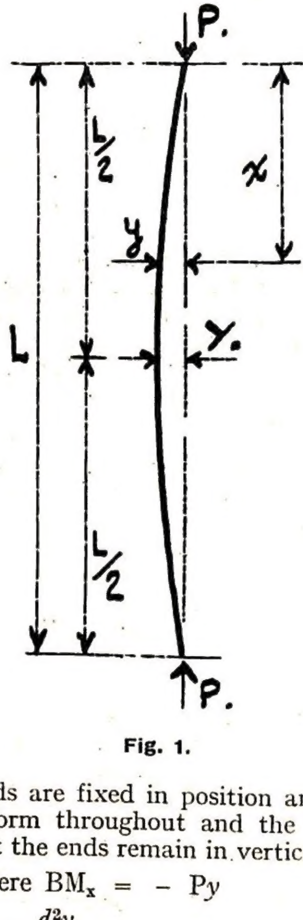


Fig. 1.

Case I.—The ends are fixed in position and are free to deflect. The section is uniform throughout and the bending moment at each end is zero, but the ends remain in vertical line.

$$\text{Here } BM_x = -Py$$

$$EI\frac{d^2y}{dx^2} = -Py$$

$$\frac{d^2y}{dx^2} + \frac{P}{IE}y = 0$$

The solution of this differential equation is

$$y = \gamma \operatorname{Sin}\left\{x\sqrt{\frac{P}{IE}} + b\right\}$$

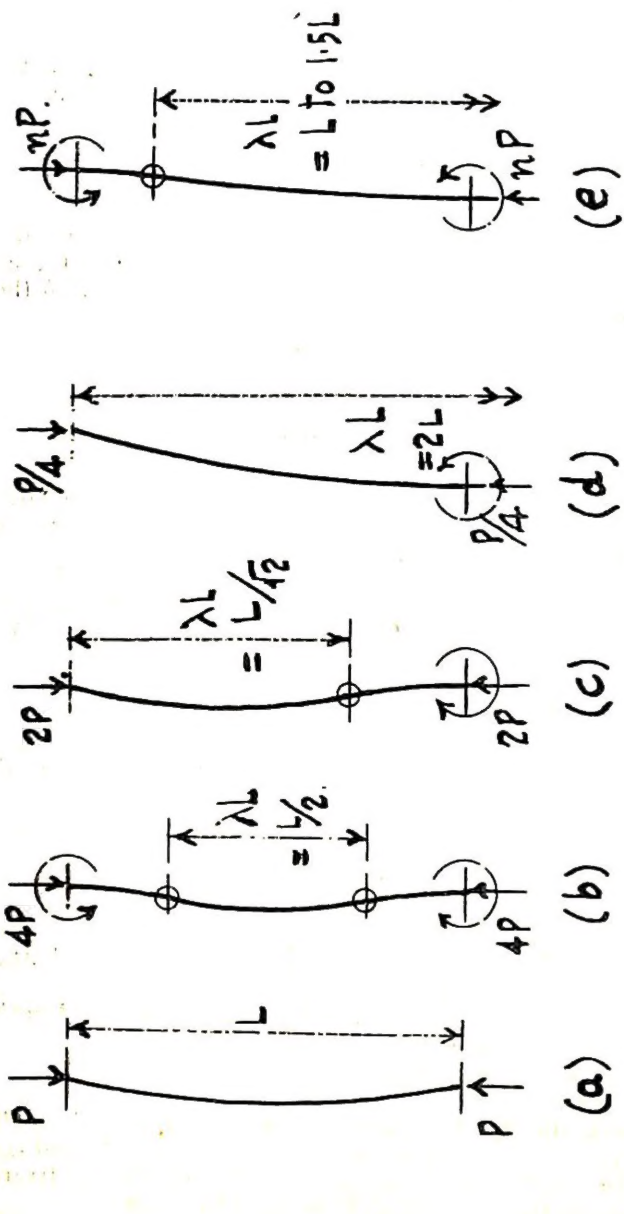


Fig. 2.

but if $x=0$, $y=0$, therefore $b=0$, and finally

$$y = \gamma \operatorname{Sin} x \sqrt{\frac{P}{IE}}$$

The maximum value of y occurs when the sine has maximum value, that is when $\operatorname{Sin} x \sqrt{\{P/IE\}} = 1{\cdot}0$ and then, since at this time $x = L/2$

$$\frac{L}{2}\sqrt{\frac{P}{IE}} = \frac{\pi}{2}$$

from which $P=\pi^2 IE/L^2$ and P is the buckling or critical load. Now $I=Ar^2$ where r is the radius of gyration about the axis of bending, usually the axis having the least value of r, and A the area of the cross section, so that

$$\begin{aligned} P &= \pi^2 A r^2 E/L^2 \\ &= \pi^2 AE/\{L/r\}^2 \end{aligned}$$

$$\frac{P}{A} = \frac{\pi^2 E}{\{L/r\}^2}$$

which is Euler's value for the crippling stress in the standard case.

In Figure 2, (a) illustrates the standard case while (b) (c) and (d) show the other cases usually considered, these being :—

Case II—(b).—The ends are position and deflection fixed.

That part λ behaves simflarly to the strut in Case I. and $\lambda=L/2$.

$$\begin{aligned} P_{ii} &= \pi^2 IE/\lambda^2 = 4\pi^2 IE/L^2 \\ &= 4P \end{aligned}$$

Case III—(c).—One end is round, the other fixed. The ends are fixed in position but one is deflection free while the other is deflection fixed.

Here λ is approximately $= 0{\cdot}7L$ $= L/\sqrt{2}$

$$\begin{aligned} P_{iii} &= 2\pi^2 IE/L^2 \\ &= 2P \end{aligned}$$

Case IV—(d).—The top has side freedom. The bottom is position and deflection fixed.

The strut is analogous to another strut having a length 2L, that is, $\lambda = 2L$.

$$\begin{aligned} P_{iv} &= \pi^2 IE/4L^2 \\ &= P/4. \end{aligned}$$

These results can be represented by the formula $P=n\pi^2 IE/L^2$ where n is a constant depending on the nature of the end connections. In the design of columns the value of the end fixation is usually one of the most difficult elements to assess.

Euler's formula has evident drawbacks since no account is taken of the direct stress P/A, where A is the section area, and the bending stress $\pm$ BM/Z caused by the unintentional eccentricity of loading. Following from Euler's deductions, the l/r value would appear to be the natural basis for assessing the strength of struts but experimental work on struts of differing sections having the same sectional areas and l/r values, has confirmed that failure occurs at different stresses. In Figure 7, the maximum combined stress on the concave side is $f = f_a + f_b$ where f_a = direct stress and f_b = stress due to eccentricity of loading.

$$\text{Hence } f = f_a + \frac{Ped}{Ar^2} \text{ since } Z = Ar^2/d$$

$$\text{or } f = f_a \{1 \div ed/r^2\}.$$

In struts having the same sectional area f will be least, and therefore the strut of greatest strength occur, when ed/r^2 is a minimum. Since inherent eccentricity is accepted as something over which there is no control, it follows that f is least when d/r^2 and therefore d/r is least. Shape of section therefore has some influence on the strength of struts. The writer's opinion is that the least possible value of d/r is about 1·2 and that struts which approach this value of d/r will be economical. It should be noted, however, that there are two principal axes and two d/r values. The axis having greatest d/r value (usually the axis having least r value) moves parallel to itself and therefore the *greatest* d/r value is the critical value so far as this indication of economy, or alternatively, carrying capacity per unit of sectional area is concerned. See comparisons in tables, page 24.

The formula $f = f_a \{1 + ed/r^2\}$ which gives

$$f_a = f / \{1 + ed/r^2\}$$

is quite rational. It holds for all conditions and when e is a definitely known intentional eccentricity.

Gordon-Rankine Formula.—This empirical formula, known in Germany as the Schwartz-Rankine formula, is obsolete so far as structural work in steel is concerned. It represents an attempt to combine the direct stress at low l/r values with Euler's stress at high values of l/r and is :—

$$P = \frac{Af_c}{1 + \frac{1}{n}\,\frac{f_c}{\pi^2 E}\left(\frac{L}{r}\right)^2}$$

where P is the ultimate load,

f_c the crushing strength, and n a constant equal to one for free ends and four for fixed ends.

The values of f_c and f_c/π^2E for various materials are :—

Material	f_c (Lbs/Sq. in.)	f_c/π^2E
Cast Iron	80,000	1/1600
Wt. Iron	36,000	1/9000
Mild Steel	48,000	1/7500
Timber (Hard)	7,200	1/750

With a proper choice of constants the formula is reliable and useful when materials other than steel, say timber or cast iron, are being considered as struts.

In a mild steel strut having free ends, taking f_c as 21 t/sq. in.

$P = 21/\{1 + 1/7500\ (l/r)^2\}$ tons/sq. in. and some calculated values are given on the graph in Figure 3 which is typical of most curves in strut formulae. Experimental results confirm the shape of the curve, as the plotted ultimate loads lie roughly above and below a graph of the type shown.

So-called straight line formulae are therefore erroneous.

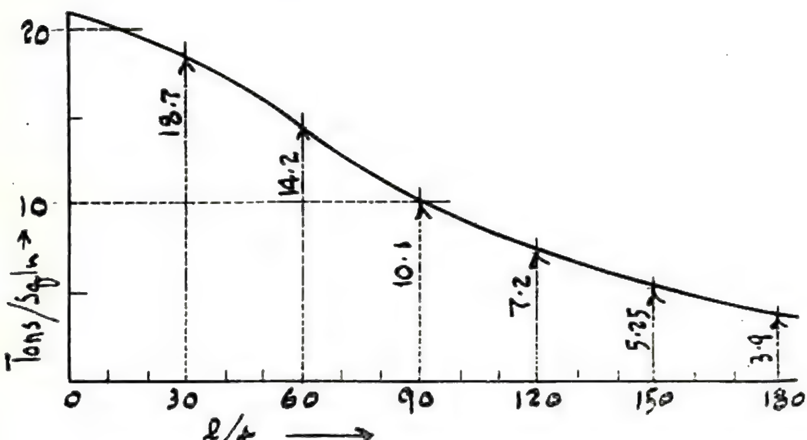


Fig. 3.

In the practical strut the l/r range is from 50 to 150 and the direct and bending stresses are equally important. A column formula should vary continuously from the crushing stress at very short lengths to the flexural (Euler's) stress at long lengths.

Ayrton-Perry Formula.—In a practical strut an initial bending moment is created by the eccentricity due to defects and this moment can be allowed for by assuming that the strut has an initial deflection curve.

Perry assumed such a curve and from its equation derived a stress formula for the fundamental case of both ends round, viz. :—

$$f = \frac{f_1 + (n+1) f_e}{2} - \sqrt{\left\{ \frac{f_1 + (n+1) f_e}{2} \right\}^2 - f_1 f_e} \quad \text{tons/sq. in.} \qquad \text{(I)}$$

where f = P/A = Ultimate gross stress.

f_1 = Compressive stress on concave side at centre of strut.

f_e = Euler's stress $= 13{,}000\pi^2/(l/r)^2$ tons/sq. in.

n = $0{\cdot}003\ l/r$.

The modifications to this formula proposed by Prof. Robertson were adopted by the British Standards Institution as the standard formula for specification No. 449. The standard formula is

$$K_2F_a = \frac{f_y + (n+1) f_e}{2} - \sqrt{\left\{ \frac{f_y + (n+1) f_e}{2} \right\}^2 - f_y f_e} \quad \text{tons/sq. in.} \qquad \text{(II)}$$

where K_2 = 2·0. *

F_a = Permissible gross stress.

f_y = Guaranteed minimum yield stress = 15·25 tons/sq. ins.

Collapse is assumed to occur when the compressive stress on the concave side (f_1 in I) reaches the yield stress (f_y in II). If, therefore, f_1 in I is given the value of the yield stress ($f_y = 15{\cdot}25$ tons/sq. in.) the result is that K_2F_a is the ultimate *end* stress in tons/sq. in. This ultimate end stress is the only stress known with reasonable accuracy since it is equal to P/A where P is the external load and A the section area.

Load Factor.—K_2 is a *load factor* and division by the value 2·0 taken, gives the working stress F_a in tons/sq. in. The use of a load factor instead of a factor of safety is increasing in constructional work and is the ratio

$$\frac{\text{Collapse End Load}}{\text{Permissible End Load}} = \frac{\text{Ultimate End Stress}}{\text{Permissible End Stress}}$$

and is *not* the ratio.

$$\frac{\text{Ultimate End Stress}}{\text{Maximum Fibre Stress}}$$

If various values are given to f_1 in Equation I of the Perry formulae, the corresponding stress f, which is the compressive stress

*The B.S.S., No. 449 (1937), took $K_2 = 2{\cdot}36$ and $f_y = 18{\cdot}00$ tons/sq. in. The new values give slightly greater permissible stresses.

at the end of the strut can be calculated. For the value $l/r = 100$ in I the calculated figures for three values of f_1 are :—

f_1	3·0 T/Sq. in.	9·0 t/sq. in.	15·25 t/sq. in. ($=f_Y$). (Guaranteed Yield Pt.).
f	2·2 ,,	5·8 ,,	8·26 t/sq. in. (End Ultimate Stress).

These figures show that f_1 increases more rapidly than f and this is accounted for by deflection increasing more rapidly than load.

When the yield stress has been reached at the centre of the strut the ultimate end stress is 8·26 tons/sq. in. The permissible end stress is then $8{\cdot}26/K_2$ or $F_a = 8{\cdot}26/2{\cdot}0 = 4{\cdot}13$ tons/sq. in. and this is the stress which will be found in the table of permissible stresses for $l/r = 100$, both ends round.

A table of permissible stress values against l/r values is given below. Intermediate values may be obtained by interpolation.

l/r	F_aT/Sq. in.	l/r	F_aT/Sq. in.	l/r	F_aT/Sq. in.
10	8·51	54	6·38	98	4·23
12	8·42	56	6·28	100	4·13
14	8·32	58	6·19	102	4·04
16	8·23	60	6·09	104	3·94
18	8·13	62	5·99	106	3·85
20	8·03	64	5·89	108	3·76
22	7·93	66	5·79	110	3·67
24	7·83	68	5·70	112	3·59
26	7·73	70	5·60	114	3·51
28	7·63	72	5·50	116	3·43
30	7·54	74	5·41	118	3·35
32	7·44	76	5·31	120	3·26
34	7·35	78	5·22	130	2·89
36	7·25	80	5·12	140	2·57
38	7·16	82	5·02	150	2·30
40	7·06	84	4·92	160	2·06
42	6·96	86	4·82	170	1·86
44	6·86	88	4·72	180	1·68
46	6·76	90	4·62	190	1·52
48	6·67	92	4·52	200	1·39
50	6·57	94	4·42	210	1·27
52	6·47	96	4·33	220	1·17

Formula II does not apply between the values $l/r = 0$ and $l/r = 80$. Within this range the variation is taken to be linear with $F_a = 9{\cdot}0$ tons/sq. in. at $l/r = 0$ and 5·12 tons/sq. in. at $l/r = 80$. The graph is shown in Figure 4 and that portion in dotted line illustrates the curve if formula II did apply.

It is usual not to exceed $l/r = 150$ for important members. Secondary members may have an l/r value greater than 150.

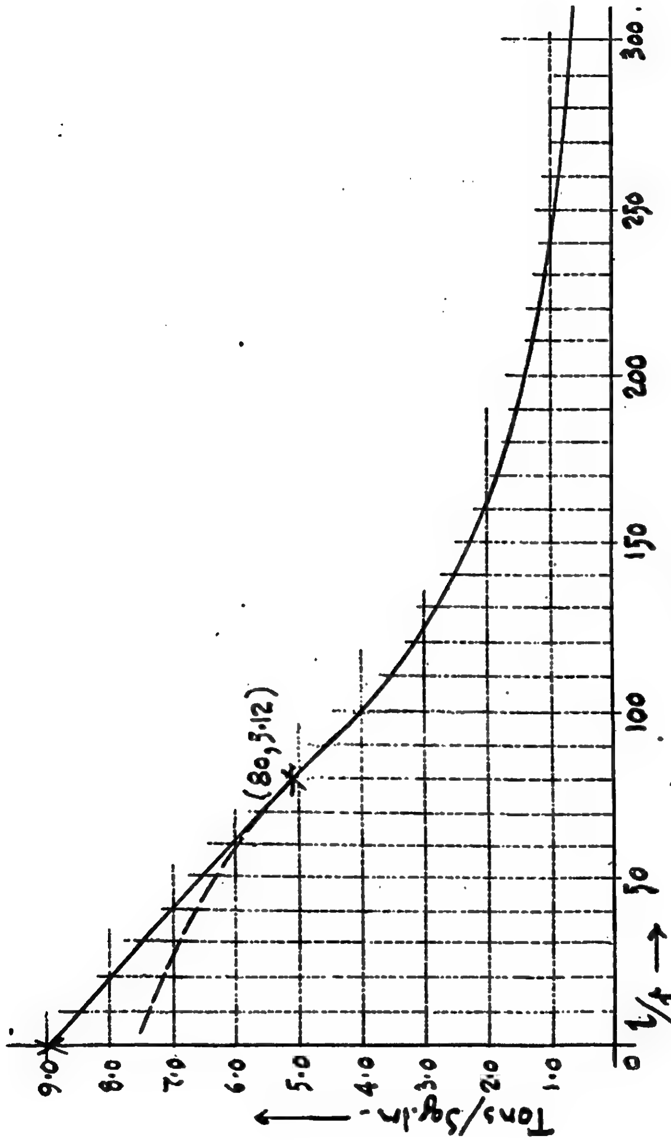

(80, 5.12)
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
50
100
150
200
250
300
l/r →
Tons/Sq.in. →

Fig. 4.

An approximation to the Perry formula, for l/r values between 80 and 140, which is of the Gordon-Rankine form is

$$f = \frac{9 \cdot 66}{1 + \frac{1}{7326}\left(\frac{l}{r}\right)^2} \text{ tons/sq. in.}$$

and may be useful instead of Perry's formula if differentiation is required.

Comparative values in tons/sq. in. are :—

l/r	f (Approximation).	F_a (B.S.S. 449).
80	5·16	5·12
100	4·08	4·13
120	3·26	3·26
140	2·62	2·57

Effective Length Factor.—If the ends are other than round the adjustment is to the l/r ratio and the effective length is a proportion of the actual length, depending on the degree of fixation. Thus for fixed ends the factor is 0·7; not 0·5, and the ultimate load is not 4 times the round end load which Euler's formulae might lead one to expect. In a practical strut this cannot be, although the ratio

$$\frac{\text{Stress for length } = \text{ L}}{\text{Stress for length } = \text{ 2L}}$$

does move towards the value 4, due to Euler's values being more and more approached as length increases.

The factor 0·7L is always taken for fixed ends and is merely an approximation, the actual points of contraflexure on a strut being dependent on the degree of fixation by the end restraints. If truly fixed ends could be attained the factor would be 0·5. The British Standard Specification, No. 449 (1937), para. 16 (a), gives some guidance regarding effective length factors. To save space these are tabulated below along with the corresponding factors in the London County Council requirements.

Table of Effective Length Factors.

Case	Top		Bottom		Eff. Length Factors	
	Position	Deflection	Position	Deflection	B.S.S. 449	L.C.C.
I. Fig. 2 (*a*)	Fixed	Free	Fixed	Free	1·0L	1·0L
II. ,, 2 (*b*)	Fixed	Fixed	Fixed	Fixed	0·7L	0·75L
III. ,, 2 (*c*)	Fixed	Free	Fixed	Fixed	0·85L	0·75 – 1·0L
IV. ,, 2 (*d*)	Free	Free	Fixed	Fixed	2·0L	2·0L
V. ,, 2 (*e*)	Free	Fixed	Fixed	Fixed	1·0L – 1·5L	

Definition of Actual Length.—According to B.S.S. 449 (1937), the actual length of a compression member shall be measured between the centres of lateral support. In the case of a compression member provided with a cap or base the point of lateral support shall be assumed to be in the plane of the top of the cap or the bottom of the base.

When column strengths are being investigated the table of stresses against l/r values is used. In general the method is :—

1. Select a trial section.
2. Calculate the l/r ratio.
3. Calculate the actual compressive stress f_a.
4. Check f_a against the permissible stress F_a.

and in a particular case, when a strut 12 ft. long with round ends is required to carry an axial load of 30 tons the procedure would be as follows :—

Try 10 in. × 5 in. R.S.J. $\quad l/r = \dfrac{144}{1 \cdot 05} = 137.$

$a = 8 \cdot 85$ sq. ins.

$r = 1 \cdot 05$ in. $\quad f_a = \dfrac{30}{8 \cdot 85} = 3 \cdot 39$ t/sq. in.

F_a = Permissible stress for $l/r = 137$ is 2·67 t/sq. in.

Therefore the 10 in. × 5 in. R.S.J. is not sufficiently strong and will only carry $2 \cdot 67 \times 8 \cdot 85 = 23 \cdot 6$ tons which is the figure given in Tables of Safe Loads.

Either l/r must be decreased in order that working stress is increased or area must be increased. Usually both take place.

Try 8 in. × 6 in. R.S.J. $\quad l/r = \dfrac{144}{1 \cdot 38} = 104.$

$a = 10 \cdot 3$ sq. ins.

$r = 1 \cdot 38$ in. $\quad f_a = \dfrac{30}{10 \cdot 3} = 2 \cdot 91$ t/sq. in.

F_a = Permissible stress for $l/r = 104$ is 3·94 t/sq. in.

This section is therefore suitable.

If the ends could be fixed the effective length would be $144 \times 0 \cdot 7 = 101$ inches, and it is interesting to see the result of such fixation.

Try 6 in. × 5 in. R.S.J. $\quad l/r = \dfrac{101}{1 \cdot 11} = 91.$

$a = 7 \cdot 37$ Sq. ins.

$r = 1 \cdot 11 \quad f_a = \dfrac{30}{7 \cdot 37} = 4 \cdot 07$ t/sq. in.

F_a = Permissible stress for $l/r = 91$ is 4·57 t/sq. in.

The 6 in. × 5 in. R.S.J. is of sufficient strength but if used the top and bottom details *must* be such that the ends can be reckoned fixed. This raises the question as to what constitutes a fixed end and the B.S.S., No. 449 (1937), can be taken as a guide in this respect. Paragraph 19 (*c*) taken verbatim from this specification reads :—

19 (*c*) Definition of a Restrained End—(i) In Direction—A pillar or other compression member may generally be assumed to have its end restrained in direction provided that the resistance moment of the restraining member (or members) and its connection is equal to :—

(*a*) 0·25 of the resistance moment of the compression member (calculated as a beam with an extreme fibre stress of 8 tons per sq. in. in the case of mild steel and 12 tons per sq. in. in the case of high tensile steel) for values of l/r not exceeding 120.

(*b*) $0{\cdot}25 + 0{\cdot}020\ (l/r - 120)$ of the resistance moment for values of l/r exceeding 120, where l = the effective length and r = the radius of gyration of the member about the axis under consideration.

(ii) Continuous Members—To constitute a restrained end condition for a compression member which is continuous through a point of lateral support, the moment of resistance of the restraining member (or members) and its connection shall not be less than half of the respective values specified in (i) (*a*) and (*b*) above.

For a compression member to be regarded as continuous through a splice, the moment of resistance at the cross section of the splice shall be not less than that specified in (i) (*a*) and (*b*) above.

With regard to the last paragraph, 50% of the resistance moment will give better practical results than the 25% recommended, and in view of the higher allowable stress in B.S.S. 449 (1948), the extreme fibre stress of 8 tons/sq. in. in paragraph 19 (*c*) (*a*) could be increased to 10 tons/sq. in. in the case of mild steel.

Paragraph 19 (*c*) implies, and is correct to imply, that the actual *number* of beams framing into a node on a column does not influence the length factor. It is conceivable that four beams at right angles having weak end connections could effect much less restraint than two beams at right angles having good end connections, and this consideration effectively disposes of a rough and ready assessment of length factors from the number of beams and which were known as one-way, two-way, three-way or four-way depending on the number of beams.

Limitations to Perry's Formula.—Perry's formula takes account of unintentional eccentricity of load and does not apply when

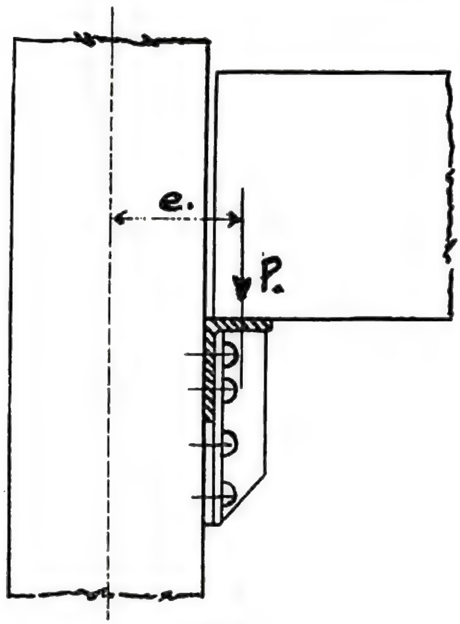


Fig. 5.

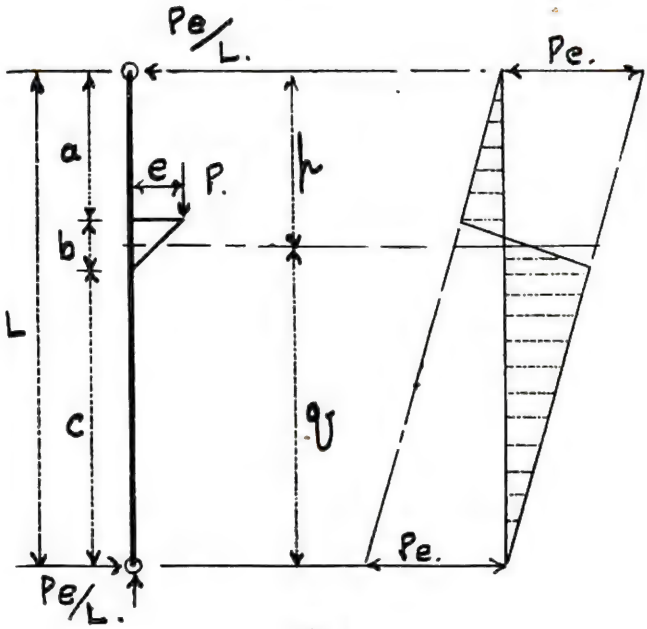


Fig. 6.

(1) the loading is intentionally off the axis; or
(2) when the loading is laterally applied.

An inclined load may be resolved into vertical and horizontal loads.

Typical of loading parallel to the axis is that of a beam which is supported by a stiffened angle on the face of a column, Figure 5. Here the eccentric moment is P*e* but this is not necessarily the bending moment in the column at the point of application for considering the column with bracket in Figure 6, P*e* is the eccentric moment but the maximum bending moment in the column, which for simplicity has free ends, is Peq/L, and this must be a maximum when p (or q) and b approach zero, that is, when the eccentric moment is at the top or bottom of the column.

Table 16 of B.S.S. 449 (1948) gives an indication of the actual eccentricity to be taken in certain cases and is reproduced below.

Type of Connection.	*Assumed Point of Application.*
(i) Stiffened Bracket.	Mid-point of stiffened seating.
(ii) Unstiffened Bracket.	Outer face of vertical leg of bracket.
(iii) Cleats to Web of Beam.	Face of Strut.
(iv) Cap :—	
(*a*) Beams of approximately equal span and load, continuous over the cap.	Mid-point of Cap.
(*b*) Other Cases.	Edge of stanchion towards span of beam except for roof truss bearings.
(*c*) Roof Truss Bearings.	No eccentricity for simple bearings without connections capable of developing an appreciable moment.

Notwithstanding item (iv) (*b*) of this table, the writer over a number of years has always assumed the distribution of the loading for the case illustrated in Figure 7 to be triangular giving a value to e of D/6, and then the stress distribution over the column would be trapezoidal as shown. If e became equal to Z/A the stress distribution would be triangular. The value of e in this case, however, is liable to vary with a number of factors including workmanship and relative stiffnesses of members. The actual value of e can be calculated (see appendix). If e is greater than D/6 and it could easily be greater if the beam is on a relatively short column, the calculated value should be used. The possibility of

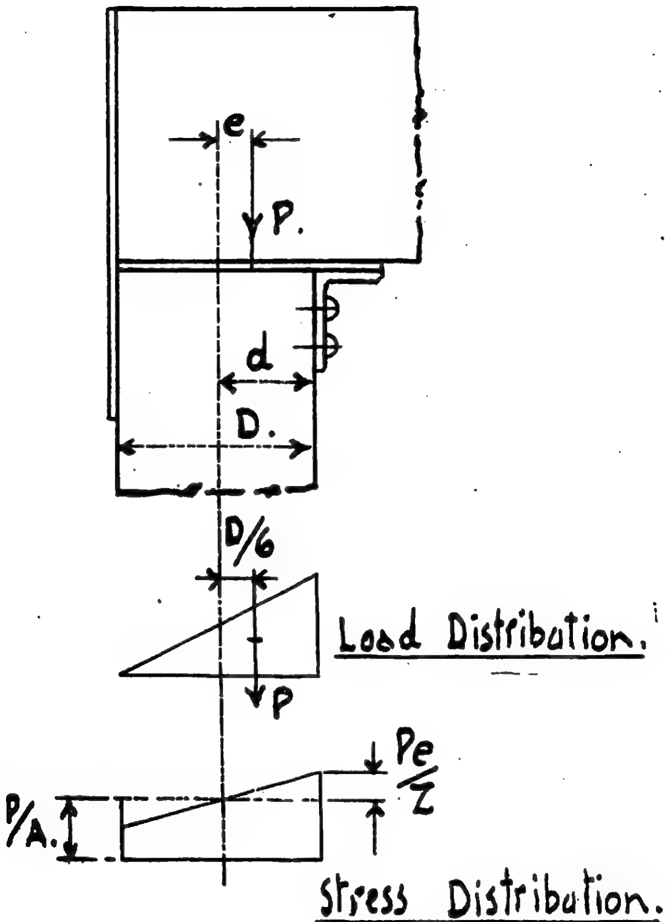


Fig. 7.

bad workmanship causing column to be other than "dead square" should not be overlooked. Column tops at incorrect levels will have the same effect.

The researches of the Steel Structures Research Committee show that the maximum stresses due to eccentricity occur at the nodes of columns where beams frame in, whereas in the normal strut concentrically loaded the maximum stress is assumed to occur at the centre. As the stresses due to eccentricity of loading become increasingly important the permissible stresses can be raised towards those allowed for beams.

The London County Council Code of Practice (1932) allowed the calculated combined direct and bending stress to be increased to

$$F_2 = f_c + 7{\cdot}5\ (1 - f_c/F_1)\ (1 - 0{\cdot}002\ l/r) \qquad \text{(A)}$$

where

f_c = Total direct stress, and

F_1 = Working stress for the appropriate l/r value calculated from the formula

$$CF_1 = \frac{p_y + (n+1)\,p_e}{2} - \sqrt{\left\{\frac{p_y + (n+1)\,p_e}{2}\right\}^2 - p_y p_e} \qquad \text{(B)}$$

where C, the load factor is 2·36

p_y = Compressive yield stress = 18 t/sq. in.

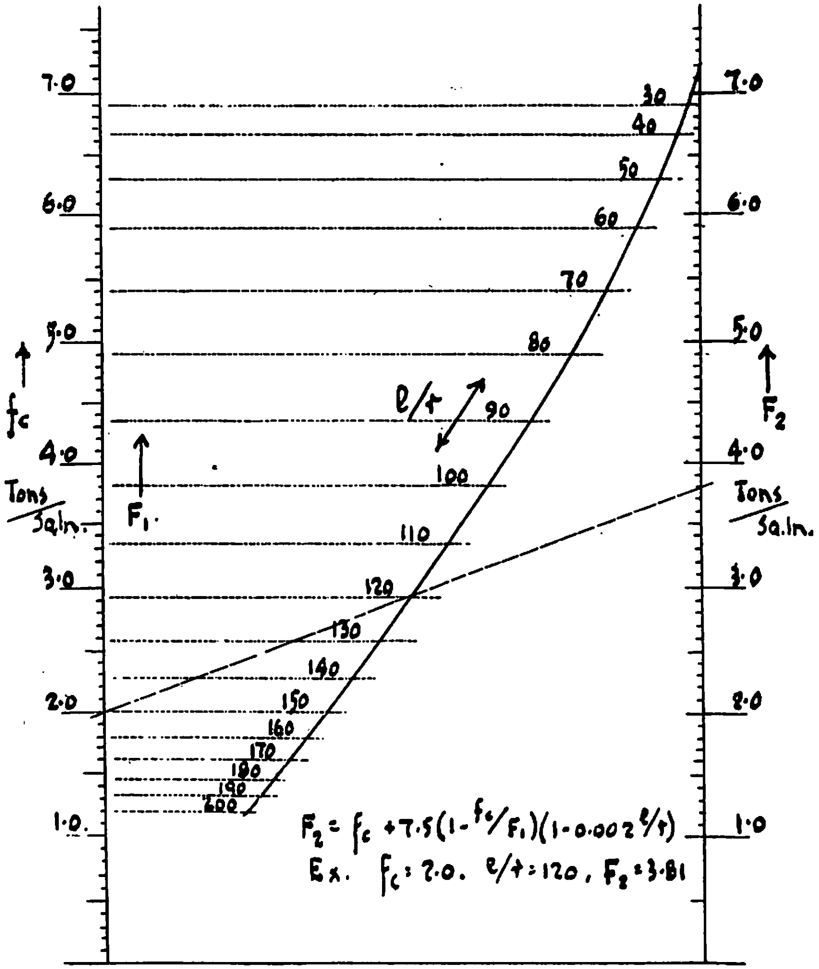
n = 0·003 l/r.

p_e = Euler's stress = $13{,}000\pi^2/(l/r)^2$.

F_1 = Permissible stress, and values calculated from Equation (B) are :—

l/r	F_1	l/r	F_1
10	7·40	110	3·34
20	7·17	120	2·93
30	6·92	130	2·58
40	6·64	140	2·28
50	6·30	150	2·02
60	5·89	160	1·81
70	5·41	170	1·62
80	4·88	180	1·46
90	4·33	190	1·33
100	3·81	200	1·21

The values of F_2 in Equation (A) may be obtained rapidly from the nomograph chart which constitutes Figure 8. Formulae (A) and (B) still apply in areas within the L.C.C. jurisdiction and are contained in "Construction of Buildings in London," published by L.C.C. (No. 3628), 1948, but the Council will be prepared to consider in respect of specific premises only, applications for modifications of the appropriate by-laws so as to permit the design of steelwork



7·0
6·0
5·0
4·0
3·0
2·0
1·0
fc
F2
Tons/Sq.In.
Tons/Sq.In.
F1
l/t
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
F2 = fc + 7·5(1 - fc/F1)(1 - 0·002 l/t)
Ex. fc = 2·0. l/t = 120, F2 = 3·81

Fig. 8.

to be based on C.P. 113 (1948) (Code of Practice for Structural Use of Steel in Buildings) and not upon B.S.S. 449, 1948.

Formulae (A) and (B) were adopted by the B.S. Committee for Specification 449 (1937), but a different formula is used in the 1948 Specification which requires (Clause 22*a*) that members subject to both axial compression and bending stresses shall be so proportioned that the quantity

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}}$$

does not exceed unity where

- f_a = actual axial compressive stress.
- F_a = the permissible compressive stress in axially loaded struts.
- f_{bc} = the sum of the compressive stresses due to bending about both rectangular axes.
- F_{bc} = the permitted maximum compressive stress for members subject to bending.

The last quantity, F_{bc}, is the allowable compressive stress in beams and clause 19 (*c*) states that F_{bc} shall not exceed :—

(i) F_{bc} = 10 tons/sq. in. for steel to B.S. 15 ; or
= $0{\cdot}65\ f_y$ for other steels, where f_y is the guaranteed yield stress in tons/sq. in.

(ii) F_{bc} = $1000\ K_1/(l/r)$ where l is the length between effective lateral restraints and r the least radius of gyration.

K_1 = a factor which shall be taken as unity except in the case of rolled steel joists, compounds and plate girders symmetrical about both principal axes and subject to bending about the *xx* axis. For these conditions the factor has values depending on the gross radii of gyration r_{xx} and r_{yy} about the principal rectangular axes :—

r_{xx}/r_{yy}	5·0	4·5	4·0	3·5	3·0 or less
K_1	1·00	1·125	1·25	1·375	1·5

intermediate values being obtained by interpolation.

Hence if $l/r = 150$ and $K_1 = 1{\cdot}5$, then $F_{bc} = 10{\cdot}0$ t/sq. in.
,, $l/r = 100$ and $K_1 = 1{\cdot}0$, then $F_{bc} = 10{\cdot}0$,, ,,

It follows that if l/r is less than 100, the value of F_{bc} is always 10·0 tons/sq. in.

Where r_{xx}/r_{yy} is less than 3·0, K_1 is 1·5 and therefore, when this applies, F_{bc} is 10·00 t/sq. in. for all values of l/r up to 150. The great majority of compound section columns have the ratio r_{xx}/r_{yy} less than 3·0 and also some R.S.J.'s, viz. :—

R.S.J.	12″ × 8″	10″ × 8″	9″ × 7″	8″ × 6″	8″ × 5″	6″ × 5″	6″ × 4½″	5″ × 4½″	4″ × 3″
r_{xx}/r_{yy}	2·72	2·29	2·28	2·42	2·96	2·20	2·53	1·94	2·44

Other rolled steel joists will require to have F_{bc} calculated when l/r is greater than 100, although F_{bc} may still be 10·00 t/sq. in. For an 8″ × 4″ R.S.J. r_{xx}/r_{yy} is 4·00 and $K_1 = 1{\cdot}25$.

If therefore $F_{bc} = 1000\ K_1/(l/r) = 10{\cdot}0.$
then $l/r = 125$, and for this particular section F_{bc} is 10·00 t/sq. in. for all l/r values up to 125.

In general, therefore, for sections which have their r_{xx}/r_{yy} ratios between 3·0 and 5·0 the l/r limit for $F_{bc} = 10{\cdot}0$ t/sq. in. is given by

$$l/r = 25\ (9{\cdot}0 - r_{xx}/r_{yy}),$$

and the tables below give l/r limit for a representative list of standard rolled steel joists.

R.S.J.	18″×8″	18″×7″	16″×8″	15″×6″	14″×8″	14″×6″ (H)
r_{xx}/r_{yy}	4 31	4·98	3·78	4·95	3·25	4·36
l/r Limit	117	100	130	101	144	116
R.S.J.	**14″×6″ (L)**	**12″×6″ (H)**	**12″×6″ (L)**	**12″×5″**	**10″×6″**	**10″×5″**
r_{xx}/r_{yy}	4·53	3·65	3·80	4·79	3·06	3·86
l/r Limit	112	134	130	105	148	128
R.S.J.	**10″×4½″**	**9″×4″**	**8″×4″**	**7″×4″**	**6″×3″**	**5″×3″**
r_{xx}/r_{yy}	4·34	4·41	4·00	3·44	3·82	3·06
l/r Limit	116	115	125	139	129	148

Values of r_{xx}/r_{yy} over 5·0 would be unusual in a column section although not impossible to obtain, for instance in an 18″ × 6″ R.S.J. this ratio would be 5·95 and the comparison of this section with a 10″ × 8″ R.S.J. which has the same weight per ft. is given in table below :—

Section D × B × w	Area in.²	Rad. of Gyr.		Ratio	Ratio		Concentric Load in Tons		
		r_{xx}	r_{yy}	r_{xx}/r_{yy}	$D/2r_{xx}$	$B/2r_{yy}$	6 Ft.	10 Ft.	16 Ft.
18″ × 6″ × 55#	16·18	7·21	1·21	5·95	1·25	**2·48**	98·8	67·5	33·8
10″ × 8″ × 55#	16·18	4·22	1·84	2·30	1·18	**2·17**	115·0	94·5	63·6

Another interesting comparison is that of two columns composed of double 6″ × 3″ channels in which one has the webs back to back; the other having the toes continuously welded. Particular attention is drawn to the ratio $(B/2)/r_{yy}$ in the light of a previous

Section D × B × w	Area in.²	Rad. of Gyr.		Ratio	Ratio		Concentric Load in Tons		
		r_{xx}	r_{yy}	r_{xx}/r_{yy}	$D/2r_{xx}$	$B/2r_{yy}$	6 Ft.	9 Ft.	12 Ft.
][6″ × 6″ × 24·8#	7·3	2·41	1·25	1·93	1·24	**2·40**	45·2	35·2	24·6
[] 6″ × 6″ × 24·8#	7·3	2·41	2·29	1·05	1·24	**1·31**	53·3	49·1	36·1

statement regarding this ratio in its relation to load carrying capacity per unit of sectional area, the box shaped column being particularly efficient. In both tables the best section to use as a strut or column is obvious.

Distribution of Moments.—Two important assumptions are made in the B.S.S. No. 449, 1948. Clause 37 (*b*) states that in effectively jointed and continuous columns the bending moments due to eccentricities of loading at any one floor may be taken as being:

(i) Ineffective at the floor levels above and below that floor.

(ii) Divided equally between the column lengths above and below that floor level, provided that the moment of inertia of either column section, divided by its actual length, does not exceed 1·5 times the corresponding value for the other length. In cases exceeding this ratio the bending moment shall be divided in proportion to the moments of inertia of the stanchion sections, divided by their respective lengths.

Experimental work does not tend to confirm the first assumption (see Figure 24). The second may be deduced by considering two fundamental cases of moment distribution.

I. When a member has a moment applied at one end, with the other end fixed but free to deflect.

In Figure 9 if M is the applied end moment the member curves as shown.

$$\text{Here IE}\ \frac{d^2y}{dx^2} = M_x = Mx/l.$$

$$\text{IE}\ \frac{dy}{dx} = Mx^2/2l + C.$$

$$\text{IE}\ y = Mx^3/6l + Cx + D.$$

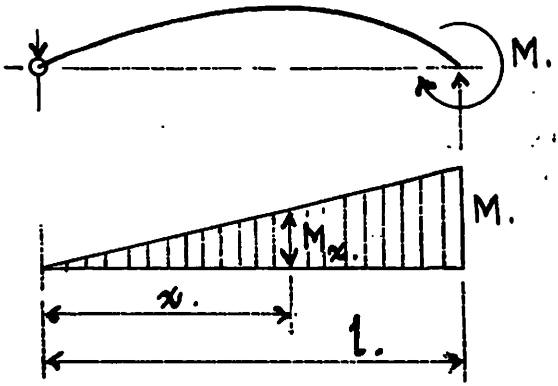


Fig. 9.

$x = 0, \quad y = 0 \quad D = 0.$
$x = l, \quad y = 0. \qquad C = -Ml/6.$

$$i.e., \quad IE\frac{dy}{dx} = Mx^2/2l - Ml/6.$$

$$\text{at end } x = l. \qquad IE\frac{dy}{dx} = Ml/2 - Ml/6 = Ml/3.$$

$$\frac{dy}{dx} = Ml/3IE \text{ for one end free to rotate.}$$

II. Fixed at one end such that that end remains horizontal. Moment at other end.

In Figure 10 (a)

$$IE\frac{d^2y}{dx^2} = M_x = M_E + \frac{M - M_E}{l}x$$

$$IE\frac{dy}{dx} = M_E x + (M - M_E)\,x^2/2l + C.$$

At $x = 0.$ $dy/dx = 0.$ $C = 0.$

$$IE\,y = M_E x^2/2 + (M - M_E)x^3/6l + D.$$

(i) $x = 0.$ $y = 0.$ $D = 0.$
(ii) $x = l.$ $y = 0.$ $D = -M_E\,l^2/2 - (M - M_E)\,l^2/6.$

If (i) and (ii) are true,

$$\text{then } M_E\,l^2/2 + Ml^2/6 - M_E\,l^2/6 = 0.$$
$$3\,M_E - M_E = -M.$$
$$M_E = -M/2.$$

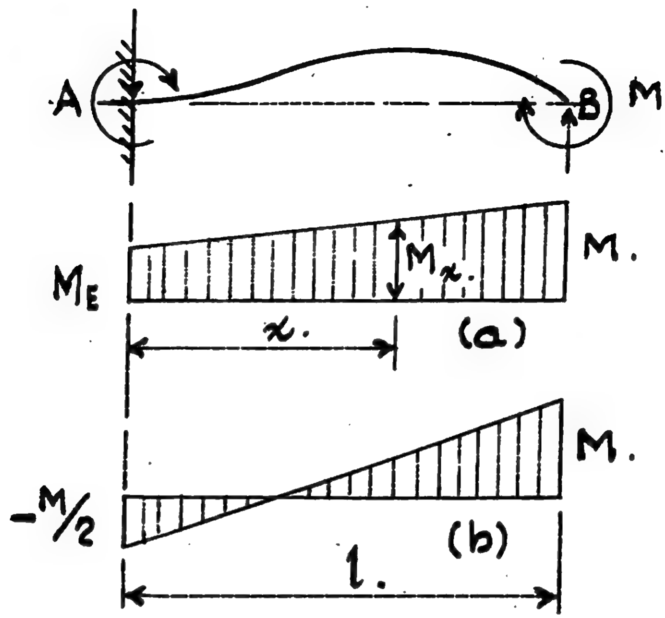


Fig. 10.

and the correct bending moment (diagram (*b*)) is shown in the figure.

At $x = l$. $\quad IE\dfrac{dy}{dx} = M_E l + (M - M_E)\, l/2.$

Put $M_E = -M/2$, the actual value,

then $\quad IE\dfrac{dy}{dx} = Ml/4.$

$$\frac{dy}{dx} = Ml/4\ IE \text{ for one end fixed.}$$

The quantity $Ml/4$ IE is taken as a measure of resistance to rotation.

Application to Columns.—Figure 11 shows two columns, one of which has free and the other fixed ends.

Whether free or fixed dy/dx has the same value on both sides of the point B at which a moment M is applied,

but $dy/dx = M_1 l_1/nI_1E = M_2l_2/nI_2E.$

where $n = 3$ for free ends
$n = 4$ for fixed ends

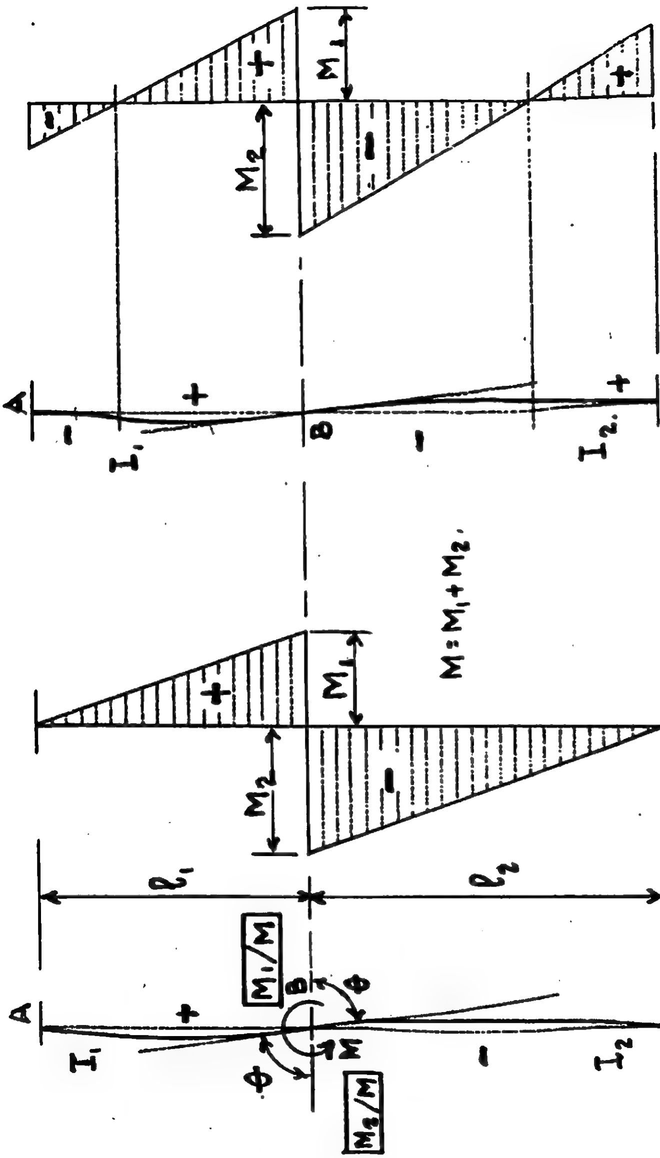


Fig. 11.

and M_1 and M_2 are the respective bending moments in each section of the column.

$$\text{Therefore } M_1/M_2 = I_1l_2/I_2l_1.$$

$$= \frac{I_1/l_1}{I_2/l_2}$$

That is, the moment M is shared in the ratio of the stiffnesses (I/l values) of the column lengths.

but $M = M_1 + M_2 = M_1 + M_1I_2l_1/I_1l_2.$

From which $M_1 = M\ (I_1/l_1)/(I_1/l_1 + I_2/l_2).$

$M_2 = M\ (I_2/l_2)/(I_1l/_1 + I_2/l_2).$

In Figure 12 the member is free at one end and fixed at the other with the moment M somewhere in between.

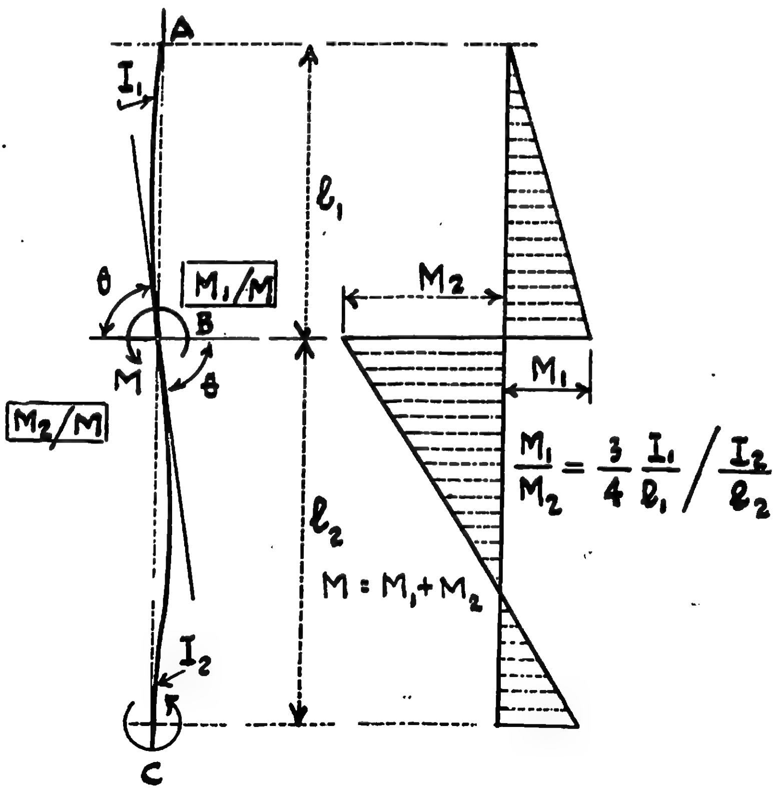


Fig. 12.

Here $dy/dx = M_1l_1/3EI_1 = M_2l_2/4EI_2$
from which

$$M_1/M_2 = \frac{3}{4}\frac{I_1}{l_1} \bigg/ \frac{I_2}{l_2}$$

and M would be divided to suit this ratio.

So far as moment distribution is concerned it is the actual length that matters, not the effective length, which has to do with effective l/r value and therefore allowable stress.

In the foregoing investigation the column nodes (A, B, C) are in line. While there is some similarity to the bending moments due to the loaded bracket on the column in Figure 6 the difference is due to the latter column being free to move laterally in the region of the bracket.

Figure 14 shows some moment distributions for the column illustrated. The factors m and n are the fractions $I_1/l_1/(I_1/l_1 + I_2/l_2)$ and $I_2/l_2/(I_1/l_1 + I_2/l_2)$ and the convention regarding $+ve$ and $-ve$ concavity and bending moments is indicated. At (*a*) the induced bending moments are assumed to be taken in the section of the column immediately below the point of application of the external moment. At (*b*) and (*c*) are shown the moment distribution for M_B with free and fixed ends respectively. At (*d*) the bending moments due to M_B are combined with those due to M_A and the distribution with fixed ends is shown in full line and in dotted line for free ends. The combination (*d*) covers the maximum possibilities and should be used as a basis for design. Many columns in the past have been designed as if the distribution is at (*a*), but in long continuing columns M_B alone without reduction by the stiffness factor may result in the lowest section, at least, being heavier than is necessary.

The actual value of the moment M_A or M_B, for the ordinary types of riveted or bolted connections, is the end reaction multiplied by the eccentricity, assumed to be in accordance with the table on page 18. If the equivalent eccentricity be calculated assuming the end connections to be rigid, it will be found that the eccentricity is very much greater than that assumed in the table. It must therefore be concluded, since this has been general practice and thousands of columns must have been designed on the assumptions stated in the table, that riveted and particularly bolted connections are such that they cannot transmit a moment anything like rigidity standard and therefore for this type of connection it is satisfactory to work to the table. The adoption of welded connections would result in greater bending moments in the columns and a corresponding relief of bending moment in the beams or girders due to the greater equivalent eccentricity of loading. A method of calculating the equivalent eccentricity in rigid connections is given in the appendix.

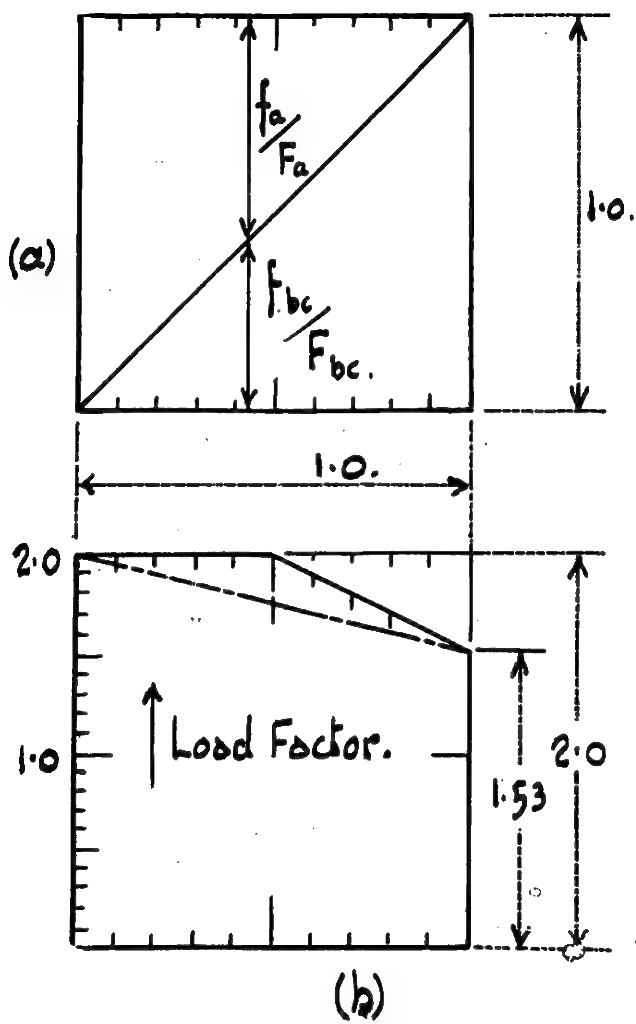


Fig. 13.

Load Factor Variation.

Taking $\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = 1$ as the maximum value of the combination to be aimed at in designing, it is evident, that if either item on the left-hand side becomes zero then the other must be unity. Between these limits, the two vary uniformly and the variation can be represented by a diagonal in a square having unit side, Figure 13 (*a*).

If l/r is 80 and over the variation in the load factor is shown at (*b*). When $f_a/F_a = 1$ the load factor according to the standard formula is 2·0 and this value is set up at (*b*) under the corresponding value of f_a/F_a. When $f_{bc}/F_{bc} = 10/10 = 1{\cdot}0$, the value of the load factor based on a yield stress of 15·3 tons/sq. in. would be 1·53 which is set up where shown and presumably the load factor would vary uniformly between the two points joined by the dotted line. The writer considers that in the region of f_a/F_a from 1·0 to 0·5 the load factor should not be less than 2·0. A constant load factor of 2·0 could be obtained by taking F_{bc} equal to 7·65 tons/sq. in. in this range. Thereafter the value of F_{bc} to be increased according to the following table.

f_a/F_a	1·0 to 0·5	0·4	0·3	0·2	0·1	0·0
f_{bc}/F_{bc}	0·0 to 0·5	0·6	0·7	0·8	0·9	1·0
F_{bc}	7·65	8·14	8·59	9·06	9·53	10·0
Load Factor	2·0	1·91	1·81	1·72	1·62	1·53

The load factors would then conform to the full line in Figure 13 (*b*).

In the range from $l/r = 0$ to $l/r = 80$ the Perry formula is not followed and therefore in this range the load factor must vary since the dotted line in Figure 4 represents the formula values for a load factor of 2·0. The actual load factors in this range may therefore be estimated by lifting the stress values from the dotted line, multiplying by 2·0 and dividing by the corresponding F_a value. Some means of estimating the actual load factor is necessary as the designer must decide if a load factor below 2·0 is justified in any particular instance, while in certain structures a load factor of 3·0 or more may be necessary.

When $f_a/F_a + f_{bc}/F_{bc}$ is equal to N, the variation may be represented by the diagonal in a square of side N. The load factor to be set up where $f_a/F_a = N$, is $2\,F_a/f_a = 2/N$, and where $f_{bc}/F_{bc} = N$, the load factor to be set up is 15·3/10 N. If these points are joined, the load factor at the actual values of the fractions may be found, and Figure 17 illustrates a particular case.

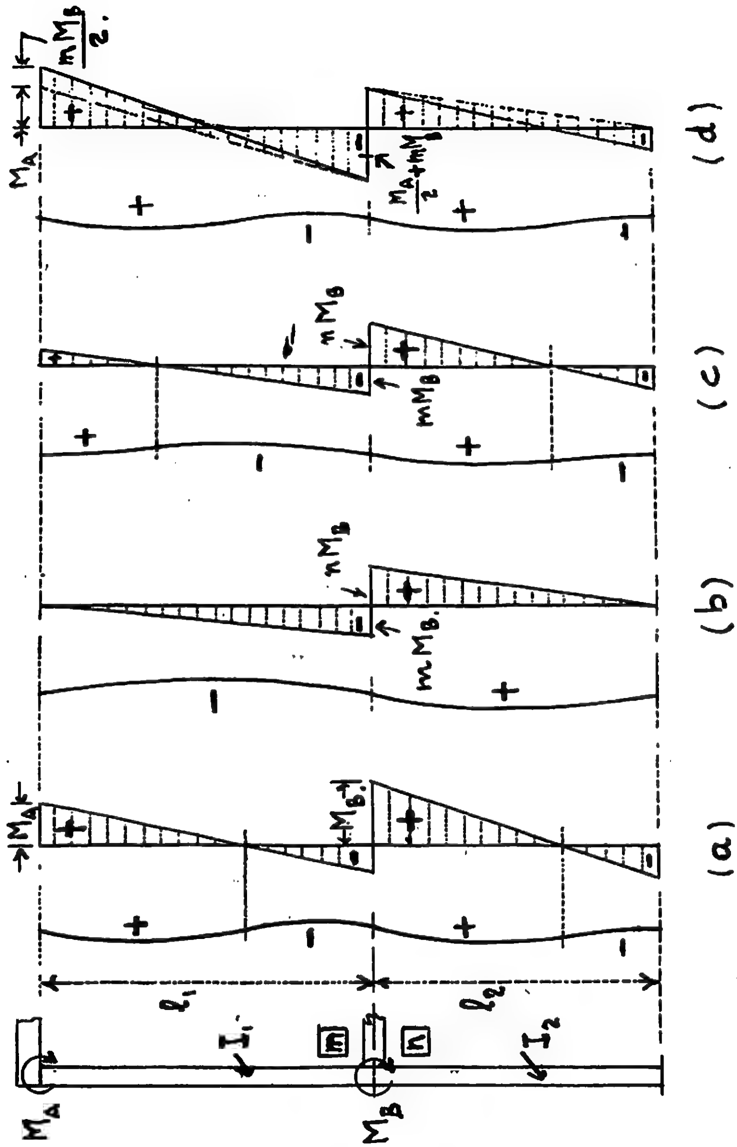


Fig. 14.

In the examples of column design which follow, total compressive stress will be denoted by f_a and the compressive stress due to bending moments about the *xx* and *yy* axes respectively by the symbols f_{xx} and f_{yy}. The sum of f_{xx} and f_{yy} will be f_{bc} to agree with B.S.S. 449 (1948). The symbol $\rightarrow$ means "produces" and all areas, section modulii, etc., in the examples will be taken from section books.

Example I.—Single Length Column.

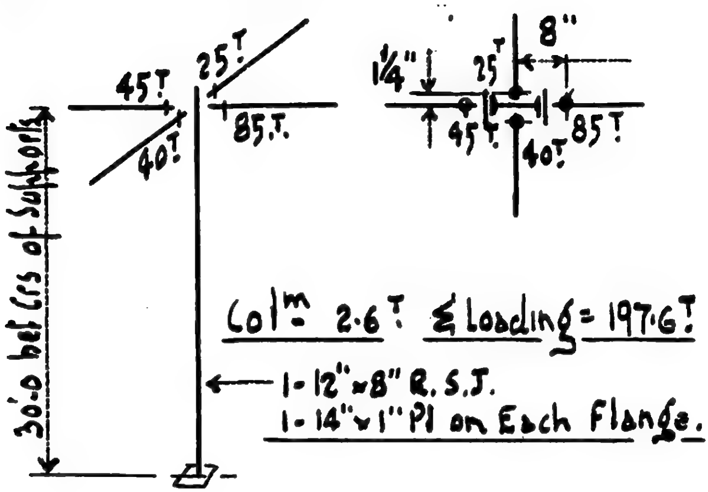


Fig. 15.

L.C.C. Code.

The column in Figure 15 has a height of 30 ft.

$$\begin{aligned}
&\text{Eff. Length} = 30 \times 0{\cdot}75 && = 22{\cdot}5 \text{ ft.}\\
&l/r = 22{\cdot}5 \times 12/3{\cdot}33 && = 81\\
&f_a = 197{\cdot}6/47{\cdot}12 && = 4{\cdot}19 \text{ t/sq. in.}\\
&f_{xx} = (85-45) \times 8/239{\cdot}0 && = 1{\cdot}34 \text{ ,, ,,}\\
&f_{yy} = (40-25) \times 1{\cdot}25/74{\cdot}6 && = 0{\cdot}25 \text{ ,, ,,}
\end{aligned}\Bigg\}\ 1{\cdot}59 \text{ t/sq. in.}$$

$$\Sigma = 5{\cdot}78 \text{ ,, ,,}$$

This maximum stress occurs immediately under the beams and for f_a ($=f_c$) and $l/r = 81$, the nomograph chart, Figure 8 gives the permissible total stress as 5·05 tons/sq. in. The column is therefore not up to L.C.C. standard.

B.S.S. 449 (1948).

$$\text{Eff. Length} = 30 \times 0{\cdot}7 = 21 \text{ ft.}$$

$$l/r = 21 \times 12/3{\cdot}33 = 76 \rightarrow F_a = 5{\cdot}31 \text{ t/sq. in.}$$

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{4{\cdot}19}{5{\cdot}31} + \frac{1{\cdot}59}{7{\cdot}65} \quad \text{(For a load factor} = 2)$$
$$= 0{\cdot}789 + 0{\cdot}208$$
$$= 0{\cdot}997.$$

The column is therefore up to B.S.S. 449 requirements.

Example II.—Double Length Column.

Figure 16 shows a column carrying a load of 230 tons including its own weight. A girder runs over the top of the column and the reaction of 75 tons from the girder will have an eccentricity equal to D/6 where D is the overall width of the column. This value should be checked to ensure that it is not exceeded. Take the 53 tons as having an eccentricity of one inch. In the absence of definite information regarding end connections take the ends as round.

It is usual in column calculations to commence with the top length and to work downwards strengthening the section as required. Nevertheless it is an advantage to obtain a rough section for the lowest length first, and in this instance using the British Steelwork Association tables for guidance, a column 20 ft. high consisting of 1 – 16″ × 8″ R.S.J. with a 14″ × 1½″ plate on each flange will carry an axial load of 346 tons (old tables). This value of the axial load allows for the eccentricity of part of the actual load and the flange thickness can be adjusted subsequently if necessary.

First Method.

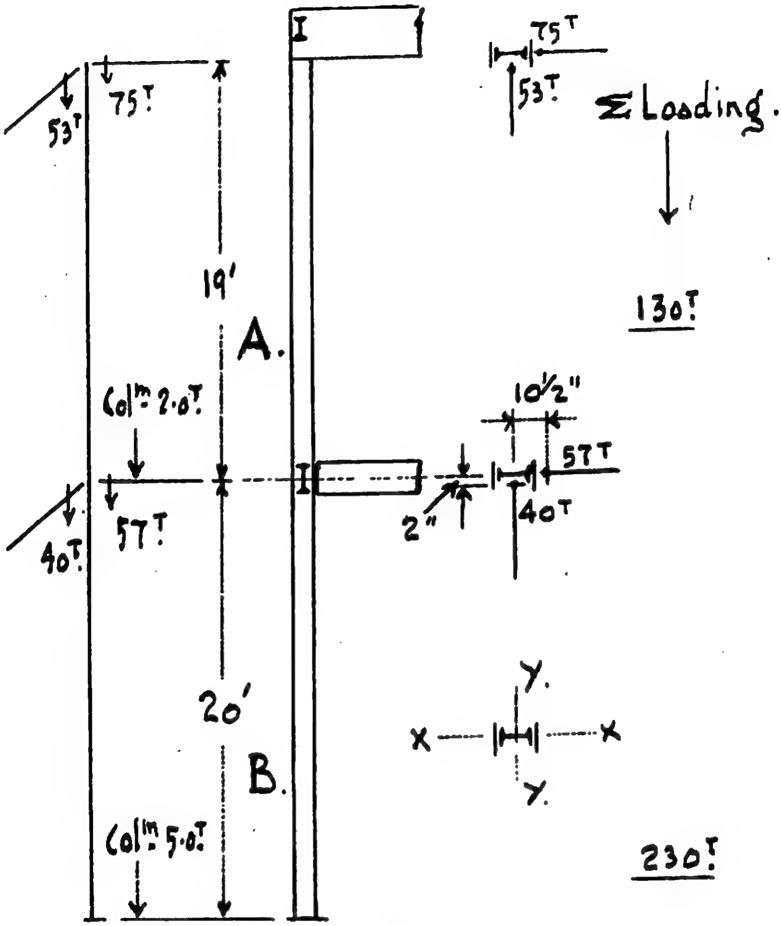
L.C.C. Section A.—Try a 16″ × 8″ R.S.J. with a 14″ × ¾″ plate on each flange.

$$\begin{aligned} l/r &= 228/3{\cdot}09 &&= 74 \\ f_a &= 130/43{\cdot}06 &&= 3{\cdot}02 \text{ t/sq. in.} \\ f_{yy} &= 75 \times 17{\cdot}5/279{\cdot}8 \times 6 &&= 0{\cdot}78 \ \text{,,} \ \text{,,} \\ f_{xx} &= 53 \times 1/58{\cdot}8 &&= 0{\cdot}90 \ \text{,,} \ \text{,,} \end{aligned} \left.\vphantom{\begin{matrix}a\\a\end{matrix}}\right\} 1{\cdot}68 \text{ t/sq. in.}$$

$$\Sigma = 4{\cdot}70 \ \text{,,} \ \text{,,}$$

$$l/r = 74 \rightarrow F_1 = 5{\cdot}20 \text{ t/sq. in.} \qquad f_a/F_1 = 0{\cdot}58.$$
$$F_2 = 3{\cdot}02 + 7{\cdot}5\ (0{\cdot}42)\ (0{\cdot}852).$$
$$= 5{\cdot}71 \text{ t/sq. in.}$$

The section is therefore satisfactory if the eccentricity of the 75 ton load does not exceed 17·5/6 = 2·9 inches. The equally distributed load on the girder being carried on the tops of the columns is 150 tons and the girder has an actual span of 40 ft. The factor F in the formula given for eccentricity in the appendix is therefore 24, and if the 2nd moment of the girder is 25,000 in^4 units the eccentricity is :—



75T
53T
19'
A.
Col^m 2.0T
57T
40T
20'
B.
Col^m 5.0T
Σ Loading.
130T
10½"
57T
2"
40T
Y.
X
X
Y.
230T

Fig. 16.

$$e = \frac{2L}{F} \quad \frac{I_H}{I_L} \quad \frac{L}{H} = \frac{2 \times 40 \times 12}{24} \quad \frac{2440}{25{,}000} \quad \frac{40 \times 12}{39 \times 12}$$
$$= 4{\cdot}00 \text{ in.}$$

This is greater than the eccentricity taken but less than Z/A given by 279·8/43·06 = 6·5 in. Therefore no tension is developed in the column.

f_{yy} corrected = 75 × 4·0/279·8 = 1·07 t/sq. in.
Max. Stress = 3·02 + 1·07 + 0·90 = 4·99 t/sq. in.

Section B.

1 – 16″ × 8″ R.S.J. with a 14″ × 1½″ plate on each flange:

l/r	= 240/3·43	= 70	
f_a	= 230/64·06	= 3·59 t/sq. in.	
f_{yy}	= 57 × 10·5/441·8	= 1·35 ,, ,,	} 2·09 t/sq. in.
f_{xx}	= 40 × 2/107·8	= 0·74 ,, ,,	
	Σ	= 5·68 ,, ,,	

Here $f_a = f_c = 3{\cdot}59$ and the permissible axial stress (F_1) for $l/r = 70$ is 5·41 T/Sq. in. Hence $f_c/F_1 = 3{\cdot}59/5{\cdot}41 = 0{\cdot}663$ and the permissible stress from $F_2 = f_c + 7{\cdot}5\,(1 - f_c/F_1)\,(1 - 0{\cdot}002l/r)$

is $F_2 = 3{\cdot}59 + 7{\cdot}5\ (0{\cdot}337)\ (0{\cdot}86)$
$= 5{\cdot}77$ t/sq. in. and is greater than the actual stress.

Second Method.

L.C.C.—In this method the moments will be shared in proportion to the I/l ratios and is more correct although more involved. The values of I/l in.³ units are :—

Section A	$I_{YY}/l = 279{\cdot}8 \times 8{\cdot}75/228 = 10{\cdot}75$	$I_{XX}/l = 58{\cdot}8 \times 7/228 = 1{\cdot}80$
,, B	$I_{YY}/l = 441{\cdot}8 \times 9{\cdot}5/240 = 17{\cdot}50$	$I_{XX}/l = 107{\cdot}8 \times 7/240 = 3{\cdot}14$
	$\Sigma = 28{\cdot}25$	$\Sigma = 4{\cdot}94$

Obviously the lower section B will be relieved of stress.

Here $l/r = 70$.

$f_c = 230/64{\cdot}06 = 3{\cdot}59$ t/sq. in.

$$f_{yy} = \frac{57 \times 10{\cdot}5}{441{\cdot}8} \times \frac{17{\cdot}5}{28{\cdot}25} = 0{\cdot}84 \text{ ,, ,,}$$
$$f_{xx} = \frac{40 \times 2}{107{\cdot}8} \times \frac{3{\cdot}14}{4{\cdot}94} = 0{\cdot}47 \text{ ,, ,,}$$

} 1·31 t/sq. in.

$\Sigma = 4{\cdot}90$,, ,,

Section A. $l/r = 74.$

	At top of Section. t/sq. in.		*At bottom of Section.* t/sq. in.
$f_a =$	3·02		3·02
$f_{yy} = \dfrac{75 \times 4}{279{\cdot}8}$	1·07	50% of 1·07	0·54
$f_{xx} = \dfrac{53 \times 1}{58{\cdot}8}$	0·90	50% of 0·90	0·45
$f_{yy} =$ 50% of 0·81 $=$	0·40	$\dfrac{57{\cdot}0 \times 10{\cdot}5}{279{\cdot}8} \times \dfrac{10{\cdot}75}{28{\cdot}25} =$	0·81
$f_{xx} =$ 50% of 0·50 $=$	0·25	$\dfrac{40 \times 2}{58{\cdot}8} \times \dfrac{1{\cdot}80}{4{\cdot}94} =$	0·50
Max. Stress $= \Sigma =$	5·64	Max. Stress $= \Sigma =$	5·32

The permissible stress calculated was 5·71 t/sq. in. Therefore section is suitable for L.C.C. requirements.

In this example the contention that loading at one node does not effect the column in the region beyond adjacent nodes has been followed. Since the stress of 1·07 tons/sq. in. is bound up with strain the writer considers that it may occur from top to bottom of the column, in which case the maximum stress in Section B would be 5·97 tons/sq. in.

Second Method.

B.S.S. 449 (1948). Section B.

$$l/r = 70 \rightarrow F_a = 5{\cdot}60 \text{ t/sq. in.}$$

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{3{\cdot}59}{5{\cdot}60} + \frac{1{\cdot}31}{10{\cdot}00} \quad \text{(Taking } F_{bc} = 10{\cdot}0 \text{ t/sq. in).}$$
$$= 0{\cdot}640 + 0{\cdot}131$$
$$= 0{\cdot}771.$$

If, however, the stress of 1·07 tons/sq. in. persists into the lower section as previously mentioned, it could be added to f_{bc} giving

$$\frac{3{\cdot}59}{5{\cdot}60} + \frac{2{\cdot}38}{10{\cdot}00} = 0{\cdot}878.$$

The section is therefore too robust.

$$\text{If } \frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = 1{\cdot}00$$

$$\frac{f_a}{F_a} = 1{\cdot}00 - 0{\cdot}238 = 0{\cdot}762.$$

$$f_a = 5{\cdot}60 \times 0{\cdot}762 = 4{\cdot}26 \text{ t/sq. in.}$$

The difference in stress is 4·26—3·59 = 0·67 tons/sq. in. and the column could therefore carry a greater *axial* load, the addition being 64·06 × 0·67 = 43 tons.

Should it be decided to reduce the section, then the approximate area required would be given by 230/4·26 = 54·0 sq. in. or a reduction of 10 sq. ins. The flange plates could be made 14″ × 1¼″ and the area of the section would then be 57·06 sq. ins., but since a reduction in area also means a reduction in section modulii the stresses would require to be calculated anew for the revised section.

Section A.

$$l/r = 74 \rightarrow F_a = 5{\cdot}41 \text{ t/sq. in.}$$

The sum of the stresses due to flexure is 2·62 t/sq. in.

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{3{\cdot}02}{5{\cdot}41} + \frac{2{\cdot}62}{10{\cdot}00}$$
$$= 0{\cdot}555 + 0{\cdot}262$$
$$= 0{\cdot}817.$$

The section is therefore too robust and should be reduced unless the extra strength is required to meet contingencies.

It will be interesting to know the value of the load factor for this section, and it may be found graphically or calculated, for if

$$\frac{f_a}{5{\cdot}41} + \frac{f_{bc}}{10{\cdot}0} = 0{\cdot}817$$

and $f_{bc} \rightarrow$ Zero. $f_a/5{\cdot}41 = 0{\cdot}817$. L.F. is 2·0/0·817 = 2·45
and if $f_a \rightarrow$ Zero. $f_{bc}/10{\cdot}0 = 0{\cdot}817$. L.F. is 15·3/8·17 = 1·87.

To find the load factor graphically; in Figure 17 take AB = 0·817 unit. Set up AC = 2·45 and BD = 1·87 to some scale. Join CD. Mark off BL = f_a/F_a = 0·555. Set up L.F. to cut CD in F, then LF is the load factor, in this instance equal to 1·87 + (2·45 − 1·87) 0·555/0·817 = 2·26.

Here l/r is below 80 but the correction to load factor on this account would be slight as the full and dotted line on graph, Fig. 4, almost coincide at l/r = 74.

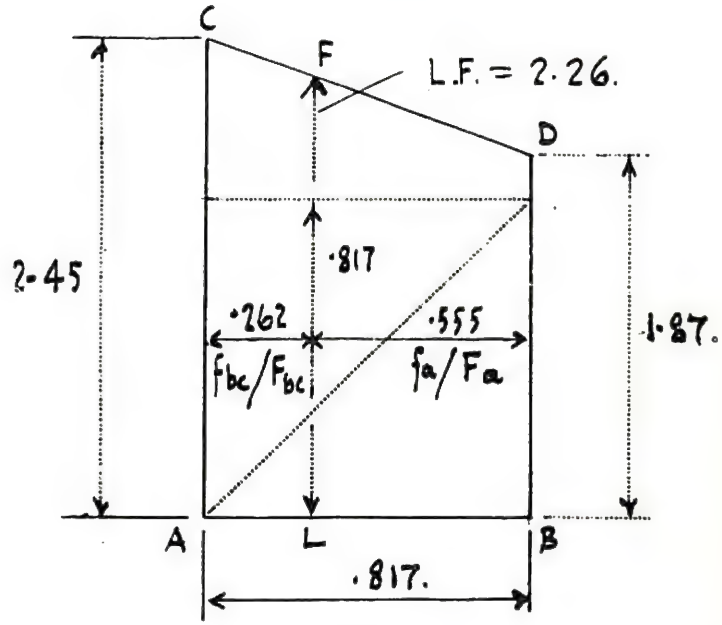


Fig. 17.

Example III.—The column shown in Figure 18 has the following particulars :—

Section.	Scantlings.	Z_{YY} in.^{3}	I_{YY} in.^{4}	I_{YY}/L in.^{3}
A. 34 ft.	2—12″×5″ R.S.Js. 2—12″×⅜ Pls.	123·4	786	1·92
B. 32 ft.	2—12″×6″ R.S.Js. 2—14″×¾″ Pls.	220·4	1485	3·86
C. 30 ft.	2—12″×6″ R.S.Js. 2—14″×1½″ Pls.	340·7	2555	7·09

The loading about the *xx* axis is balanced ; the distribution factors for the *yy* axis are :—

$$1{\cdot}92/(1{\cdot}92+3{\cdot}86) = 0{\cdot}33 \text{ and } 3{\cdot}86/5{\cdot}78 = 0{\cdot}67.$$
$$3{\cdot}86/(3{\cdot}86+7{\cdot}09) = 0{\cdot}35 \text{ and } 7{\cdot}09/10{\cdot}95 = 0{\cdot}65.$$

and these are shown in rectangles near the column at the places where they apply.

The beams will be considered to have end connections which will fix the nodes of the column. The effective length factors will therefore be 0·7.

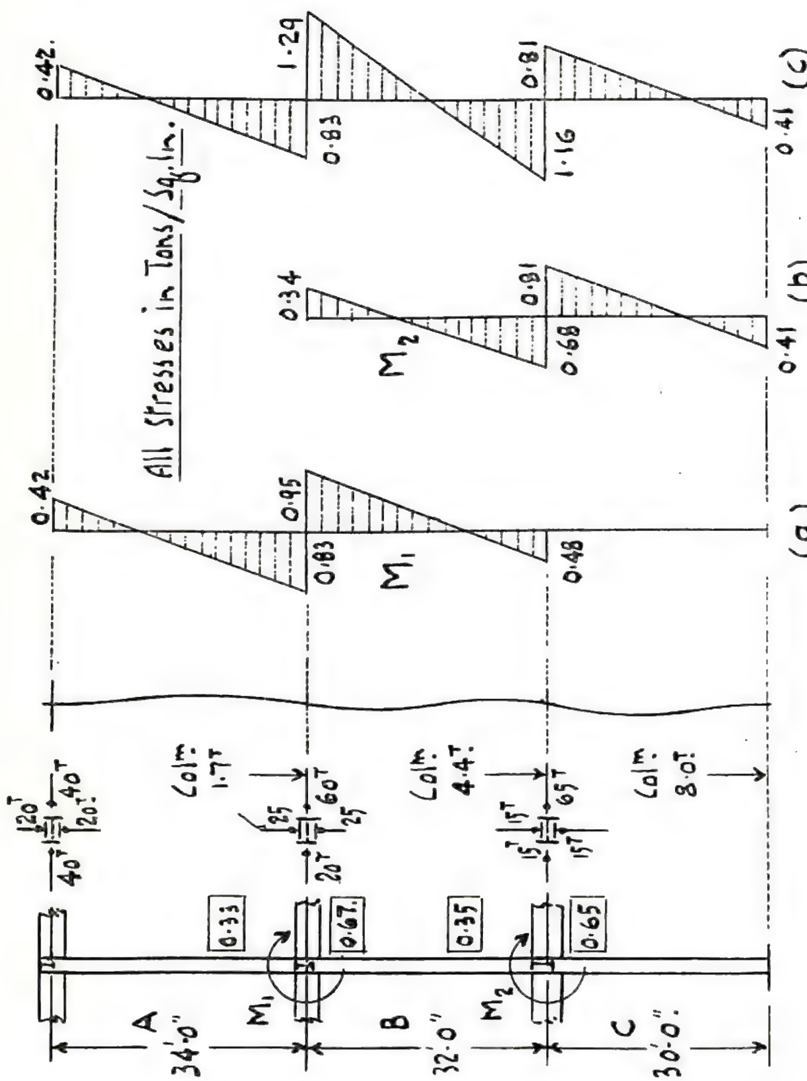

All Stresses in Tons/Sq. In.
0·42
1·29
0·81
0·83
1·16
0·41
(c)
0·34
0·81
M2
0·68
0·41
(b)
0·42
0·95
0·83
M1
0·48
(a)
Col^m: 1·7^T
Col^m: 4·4^T
Col^m: 8·0^T
120^T
40^T
20^T
40^T
20^T
25
60^T
25
20^T
15^T
15^T
65^T
15^T
15^T
0·33
0·67
0·35
0·65
A
34'·0"
M1
B
32'·0"
M2
C
30'·0"

Fig. 18.

The loading at the top of the column is balanced and hence there are no moments to consider. At (*a*) and (*b*) the stresses due to M_1 and M_2 are shown and combined at (*c*).

$$\text{Thus } M_1 = 40 \times 7{\cdot}75 = 310 \text{ in. tons.}$$
$$M_2 = 50 \times 8{\cdot}5 = 425 \text{ ,, ,,}$$

At M_1 Stress above node $= 310 \times 0{\cdot}33/123{\cdot}4 = 0{\cdot}83$ t/sq. in.
,, M_1 ,, below ,, $= 310 \times 0{\cdot}67/220{\cdot}4 = 0{\cdot}95$,, ,,
At M_2 Stress above node $= 425 \times 0{\cdot}35/220{\cdot}4 = 0{\cdot}68$ t/sq. in.
,, M_2 ,, below ,, $= 425 \times 0{\cdot}65/340{\cdot}7 = 0{\cdot}81$,, ,,

The stresses are given in lieu of the bending moments to which they are in direct proportion.

Take $F_{bc} = 10{\cdot}00$ tons/sq. in. in order that the column will be as economical as B.S.S. 449 allows.

Section A.

$$l/r = 408 \times 0{\cdot}7/3{\cdot}27 = 87 \rightarrow F_a = 4{\cdot}77 \text{ t/sq. in.}$$
$$f_a = 121{\cdot}7/27{\cdot}9 = 4{\cdot}35 \text{ t/sq. in.}$$
$$f_{yy} = 0{\cdot}83 \text{ ,, ,,}$$
$$\Sigma = 5{\cdot}18 \text{ ,, ,,}$$

$$\frac{4{\cdot}35}{4{\cdot}77} + \frac{0{\cdot}83}{10{\cdot}00} = 0{\cdot}913 + 0{\cdot}083$$
$$= 0{\cdot}996.$$

Section B.

$$l/r = 384 \times 0{\cdot}7/3{\cdot}88 = 69 \rightarrow F_a = 5{\cdot}65 \text{ t/sq. in.}$$
$$f_a = 254{\cdot}4/47{\cdot}0 = 5{\cdot}41 \text{ t/sq. in.}$$
$$f_{yy} = 1{\cdot}29 \text{ ,, ,,}$$
$$\Sigma = 6{\cdot}70 \text{ ,, ,,}$$

$$\frac{5{\cdot}41}{5{\cdot}65} + \frac{1{\cdot}29}{10{\cdot}00} = 0{\cdot}959 + 0{\cdot}129$$
$$\doteq 1{\cdot}088.$$

The flange plates should be made $14'' \times \frac{7}{8}''$ instead of $14'' \times \frac{3}{4}''$, and this increase will make the section satisfactory.

Section C.

$$
\begin{aligned}
l/r &= 360 \times 0{\cdot}7/3{\cdot}93 = 64 \rightarrow F_a = 5{\cdot}89 \text{ t/sq. in.}\\
f_a &= 368/68{\cdot}0 = 5{\cdot}40 \text{ t/sq. in.}\\
f_{yy} &= 0{\cdot}81 \quad ,, \quad ,,\\
\Sigma &= 6{\cdot}21 \quad ,, \quad ,,
\end{aligned}
$$

$$
\frac{5{\cdot}40}{5{\cdot}89} + \frac{0{\cdot}81}{10{\cdot}00} = 0{\cdot}919 + 0{\cdot}081 = 1{\cdot}000.
$$

This column as designed represents the utmost economy that can be achieved under present day practice. The load factor is just under 2·00 and whether this would be sufficient or otherwise is a matter for the individual designer and the work for which the column is intended.

The designer can easily obtain a greater load factor, if necessary, in any particular instance.

Example IV.—The column, Figure 19, is identical to that in Example III. but the moment M_2 is reversed to show the effect on flexure curve and combined stresses.

Sections A and C are as for Example III.

For Section B. $l/r = 69 \rightarrow F_a = 5{\cdot}65$ t/sq. in.

$$
\begin{aligned}
f_a &= 254{\cdot}4/47{\cdot}0 = 5{\cdot}41 \text{ t/sq. in.}\\
f_{yy} &= 0{\cdot}61 \quad ,, \quad ,,\\
\Sigma &= 6{\cdot}02 \quad ,, \quad ,,
\end{aligned}
$$

$$
\frac{5{\cdot}41}{5{\cdot}65} \times \frac{0{\cdot}61}{10{\cdot}00} = 0{\cdot}959 + 0{\cdot}061 = 1{\cdot}02.
$$

The sections could therefore be left with $14'' \times \frac{3}{4}''$ plates as originally intended.

Wind Loading.—Clause 25 of B.S.S. 449 states that the permissible stresses may be increased by 25% when wind loads occur and are taken into account, but the column section must not be less than the section required if stresses due to wind are neglected. Stresses due to wind should be added to f_a and f_{bc} and the column checked in the prescribed manner with F_a and F_{bc} increased 25%.

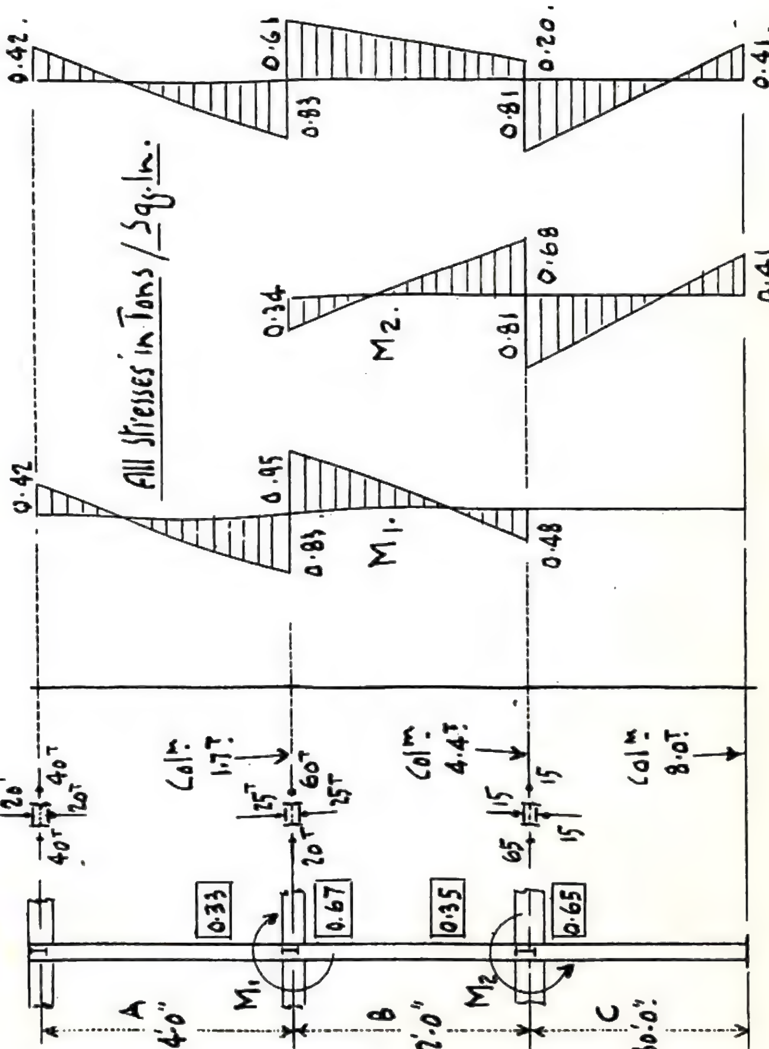

All Stresses in Tons / Sq. In.
M1.
M2.
A
34'-0"
B
32'-0"
C
30'-0"
Col.m 1.7T.
Col.m 4.4T.
Col.m 8.0T.
0.33
0.67
0.35
0.65
0.42
0.83
0.95
0.48
0.34
0.81
0.68
0.41
0.61
0.20

Fig. 19.

APPENDIX.

The slope at the end of a beam or girder is of paramount importance in the development of the subject and may be found by direct integration of the IEd^2y/dx^2 = bending moment formula. Beams which frame into columns have a bending moment at their ends of magnitude M, Figure 20, and for a central point load (*a*).

$$\text{IE } d^2y/dx^2 = Wx/2 - M.$$
$$\text{IE } dy/dx = Wx^2/4 - Mx + C.$$

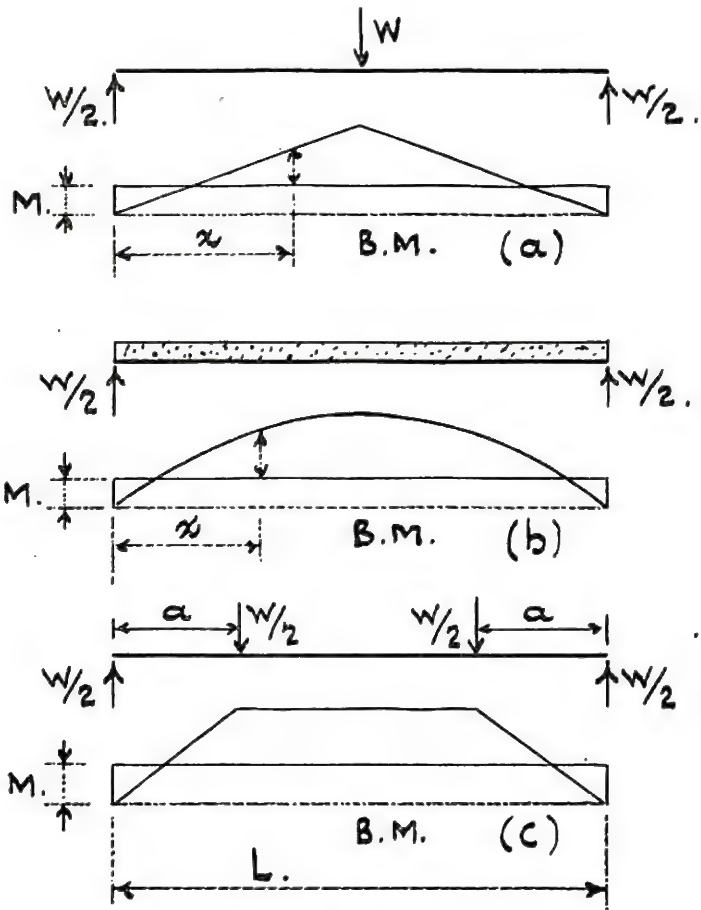


Fig. 20.

$x = L/2. \quad dy/dx = 0. \quad C = ML/2 - WL^2/16.$

$IE\ dy/dx = Wx^2/4 - Mx + ML/2 - WL^2/16.$

When $x = 0$, *i.e.*, at ends.

$dy/dx = (ML/2 - WL^2/16)\ /\ IE.$

If $M = 0. \quad dy/dx = -\ WL^2/16IE.$

For an equally distributed load (*b*)

$IE\ d^2y/dx^2 = Wx/2 - Wx^2/2L - M$

$IE\ dy/dx = Wx^2/4 - Wx^3/6L - Mx + C.$

When $x = L/2. \quad dy/dx = 0. \quad C = ML/2 + WL^2/48 - WL^2/16$

$= ML/2 - WL^2/24.$

When $x = 0. \quad dy/dx = (ML/2 - WL^2/24)\ /\ IE.$

If $M = 0. \quad dy/dx = -\ WL^2/24\ IE.$

In general, therefore,

$dy/dx = (ML/2 - WL^2/F)\ /\ IE$ where F is dependent on the nature of the loading. One other important value of F is that in the case of a beam supporting two point loads each equal to W/2, Figure 20 (*c*), and here $F = 4L^2/a\ (L - a)$.

If the girder resting on two columns in the structure illustrated in Figure 21 has a total load W symmetrically disposed it will have acquired a slope at both ends. The columns will :—

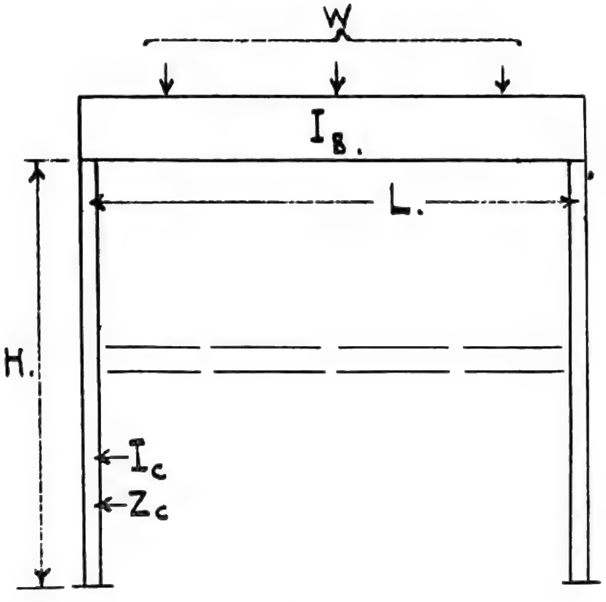


Fig. 21.

(1) Bend in a manner analogous to that of a continuous beam on four supports or

(2) Remain straight and be stressed in proportion to the strain which must occur in them.

The practical column is not usually a slender member and the writer does not consider the first of these to be correct; the columns are in direct compression and must yield. If the girder is very stiff the slope at the ends would be negligible and the strain and therefore column stress would be uniform, the stress being P/A, where P is the reaction from the girder and A the section area of the column. At (*a*) in Figure 22, this strain can be represented by MN and the stress by *mn* at (*b*). If now the girder be considered to lose stiffness the ends will slope and since the ends are considered as merely resting on the columns there is no restraining moment. The column top under the girder takes up the same slope for all or part of the width of the column and stress is increased on the girder side and decreased on the other side in proportion to the strain. Moreover, since the *average* values of the stress cannot alter, the new stress and strain lines must pass through the point 0 in each diagram. In the strain diagram the slope of OQ is given by

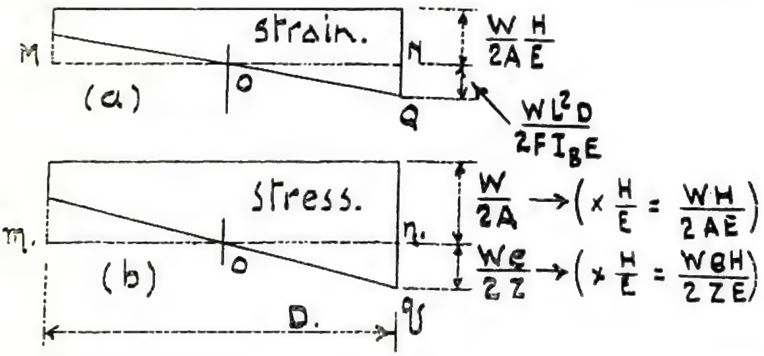


Fig. 22.

$dy/dx = WL^2/FI_BE$ where I_B is the 2nd moment of the girder.

$$\text{Hence } 2\,NQ/D = WL^2/FI_BE,$$
$$\text{or } NQ = WL^2D/2FI_BE.$$

The stress due to W/2 acting at an eccentricity e is given by $We/2Z_c$ and since the two diagrams are proportional

$$\frac{We}{2Z_c}\,\frac{H}{E} = \frac{WL^2D}{2FI_BE}$$

giving the equivalent eccentricity

$$e = \frac{2L}{F} \quad \frac{I_C}{I_B} \quad \frac{L}{H} \qquad \text{Since } Z_c = 2I_c/D.$$

where D is width of column.

This formula was used in Example II., page 36.

Whenever possible e should be such that it will not cause tension in the column. This can be ensured by altering I_C and I_B (assuming L and H fixed) until e is less than Z/A, but with long girders on short columns this may be impossible.

The connection in Figure 7, page 19, is such that it can transmit tension to the column if e is great enough, and therefore develop actual flexure in the column.

The equivalent eccentricity when a beam is on the side of a column will now be considered assuming that the connection is rigid.

In Figure 23 (a), a beam of length L frames on to a column having free ends, and at (b) the column has fixed ends.

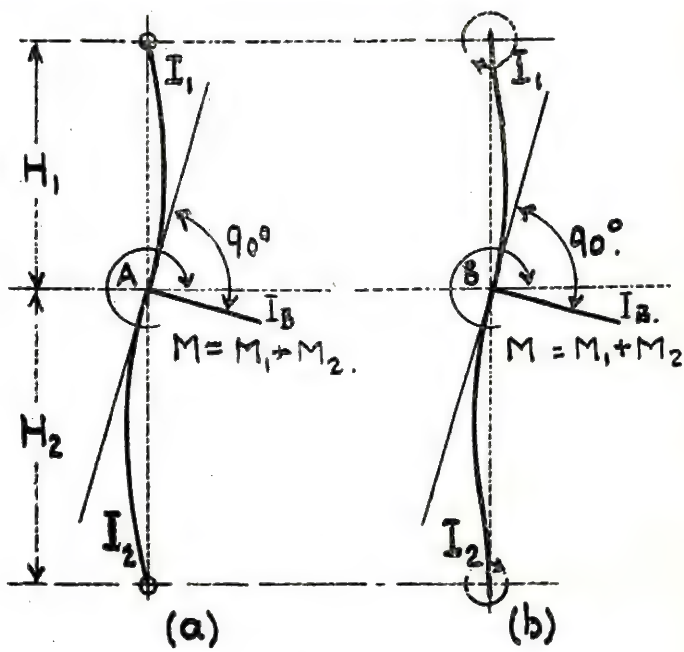


Fig. 23.

From Figure 9 and the script on page 25, it can be deduced that at A (*a*)

$$dx/dv = M_1H_1/3EI_1 = M_2H_2/3EI_2$$

but M is the end moment common to both beam and column and is equal to $M_1 + M_2$ numerically.

$$\text{Therefore } dy/dx = ML/2EI_B - WL^2/FEI_B$$
$$= M_1H_1/3EI_1 = M_2H_2/3EI_2$$
$$\text{giving } M_2 = M_1H_1I_2/I_1H_2.$$
$$(M_1 + M_2)\, L/2I_B + M_1H_1/3I_1 = -\, WL^2/FI_B$$

$$M_1\left(\frac{L}{2I_B} + \frac{L\,H_1\,I_2}{2I_B\,I_1\,H_2} + \frac{H_1}{3I_1}\right) = -\,\frac{WL^2}{FI_B}$$

$$M_2\left(\frac{L}{2I_B} + \frac{L\,H_2\,I_1}{2I_BI_2H_1} + \frac{H_2}{3\,I_2}\right) = -\,\frac{WL^2}{F\,I_B}\ *$$

so that M_1 and M_2, their sum M and hence the equivalent eccentricity can be calculated.

Similarly from Figure 10 and the script on page 26, it can be deduced that at B (*b*) in Figure 23

$$dx/dy = M_1H_1/4EI_1 = M_2H_2/4EI_2$$
$$M_2 = M_1H_1I_2/I_1H_2$$

and hence

$$M_1\left(\frac{L}{2I_B} + \frac{L\,H_1\,I_2}{2I_BI_1H_2} + \frac{H_1}{4I_1}\right) = -\,\frac{WL^2}{FI_B}$$

$$M_2\left(\frac{L}{2I_B} + \frac{L\,H_2\,I_1}{2I_BI_2H_1} + \frac{H_2}{4I_2}\right) = -\,\frac{WL^2}{FI_B}\ *$$

so that the equivalent eccentricity can be calculated. To illustrate this, one of the frames tested by the Steel Structures Research Committee will be chosen.

* If a beam at the top of a column frames on to a face the general construction is as shown in Fig. 24. The terms containing M_1 I_1 H_1 disappear and $M_2 = M$ giving :—

$$M\left(\frac{L}{2I_B} + \frac{H_2}{nI_2}\right) = -\,\frac{WL^2}{FI_B}$$

if the connection is rigid, *n* being equal to 3·0 when the remote end is free and equal to 4·0 when the remote end is fixed.

In the second example of column calculations a girder rests on the top of a column. If it had framed on to the face then :—

$$M\left(\frac{480}{2\times25000} + \frac{228}{4\times8{\cdot}75\times279{\cdot}8}\right) = -\,\frac{150\times480\times480}{24\times25000}$$

giving M=1750 in./tons and the equivalent eccentricity e=1750/75=23·2 ins

In Figure 24 the members all consist of $8'' \times 4'' \times 18$ lbs. R.S.J., which have $I_{xx} = 55 \cdot 6$ in.^{4} $= I_B$ for beam.
$I_{yy} = 3 \cdot 5$ in.^{4} $= I_1 = I_2$ for column.

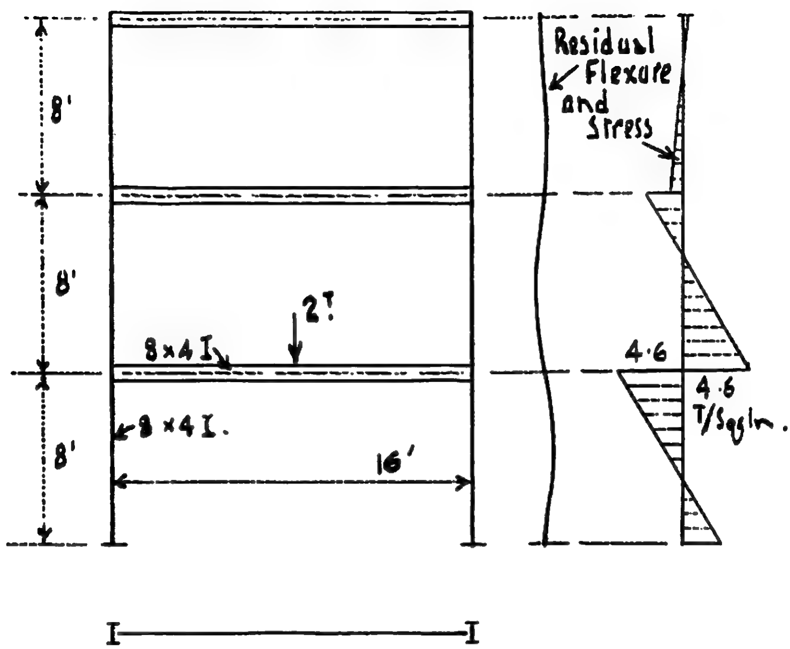


Fig. 24.

The dimensions and loading were as shown so that $H_1 = H_2 = 8$ ft. and $L = 16$ ft. Lastly for a central point load, $F = 16$ and by direct substitution in the formula for fixed ends.

$$M_1 \left(\frac{16 \times 12}{2 \times 55 \cdot 6} + \frac{16 \times 12}{2 \times 55 \cdot 6} + \frac{8 \times 12}{4 \times 3 \cdot 5} \right) = \frac{2 \times 16 \times 12 \times 16 \times 12}{16 \times 55 \cdot 6}$$

$$M_1 \, (1 \cdot 73 + 1 \cdot 73 + 6 \cdot 85) = 82 \cdot 8.$$
$$M_1 = 82 \cdot 8/10 \cdot 31 = 8 \cdot 01 \text{ in. tons.}$$
$$M = M_1 + M_2 = 2M_1 = 16 \text{ in. tons.}$$

Since the end reaction is one ton the equivalent eccentricity for this connection is $16/1 \cdot 0 = 16$ inches, which is much greater than would normally be taken and the stress in the column would be :—

$$\begin{aligned} f_{yy} &= 8 \cdot 0/Z = 8 \cdot 0/1 \cdot 75 \\ &= 4 \cdot 58 \text{ tons/sq. in.} \\ &= 10260 \text{ lbs./sq. in.} \end{aligned}$$

The actual observed stress with semi-rigid connections was about 6000 lbs./sq. in.

Columns with rigid connections therefore demand more care in their design, for while the beams are relieved of bending moment and therefore stress, this occurs at the expense of the columns. Some stiffening may be required on account of transverse stresses and the economy which seems to be expected from rigid and semi-rigid construction may not be realized.

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"The Use of Structural Steel in Building," B.S. 449 : 1948, may be obtained from the British Standards Institution, 24/8 Victoria Street, London, S.W.1, price 6/-. The price to A.E.S.D. members is 4/-, if ordered through the EDITOR, *The Draughtsman*.

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23. 1″ ,, ,, ,, ,,
24. ¾″ Sq. Steel Tubes as Struts (30 ton yield).
25. ⅞″ ,, ,, ,, (30 ton yield).
26. 1″ ,, ,, ,, (30 ton yield).
27. ¾″ ,, ,, ,, (40 ton yield).
28. ⅞″ ,, ,, ,, (40 ton yield).
29. 1″ ,, ,, ,, (40 ton yield).
30. Moments of Inertia of Built-up Sections (Tables)
31. Moments of Inertia of Built-up Sections (Instructions and Examples) } Connected.
32. Reinforced Concrete Slabs (Line Chart)
33. Reinforced Concrete Slabs (Instructions and Examples) } Connected.
34. Capacity and Speed Chart for Troughed Band Conveyors.
35. Screw Propeller Design (Sheet 1, Diameter Chart)
36. ,, ,, ,, (Sheet 2, Pitch Chart)
37. ,, ,, ,, (Sheet 3, Notes and Examples) } Connected.
38. Open Coil Conical Springs.
39. Close Coil Conical Springs.
40. Trajectory Described by Belt Conveyors. (Revised, 1949).
41. Metric Equivalents.
42. Useful Conversion Factors.
43. Torsion of Non-Circular Shafts.
44. Railway Vehicles on Curves.
45. Chart of R.S. Angle Purlins.
46. Coned Plate Developments.
47. Solution of Triangles (Sheet 1, Right Angles),
48. Solution of Triangles (Sheet 2, Oblique Angles).
49. Relation between Length, Linear Movement and Angular Movement of Lever (Diagram and Notes).
50. ,, ,, ,, ,, ,, ,, (Chart).

51. Helix Angle and Efficiency of Screws and Worms.
52. Approximate Radius of Gyration of Various Sections.
53. Helical Spring Graphs (Round Wire)
54. " " " (Round Wire) Connected.
55. " " " (Square Wire)
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57. Ratio of Length/depth of Girders for Stiffness.
58. Graphs for Strength of Rectangular Flat Plates of Uniform Thickness.
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60. Moment of Resistance of Reinforced Concrete Beams.
61. Deflection of Leaf Spring.
62. Strength of Leaf Spring.
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64. Shaft Horse Power and Proportions of Worm Gear.
65. Ring with Uniform Internal Load (Tengential Strain) Connected.
66. Ring with Uniform Internal Load (Tangential Stress)
67. Hub Pressed on to Steel Shaft. (Maximum Tangential Stress at Bore of Hub).
68. Hub Pressed on to Steel Shaft. (Radial Gripping Pressure between Hub and Shaft).
69. Rotating Disc (Steel) Tangential Strain Connected.
70. " " " Stress
71. Ring with Uniform External Load, Tangential Strain Connected.
72. " " " " " Stress
73. Viscosity Temperature Chart for Converting Commercial to Absolute Viscosities.
74. Journal Friction on Bearings. Connected.
75. Ring Oil Bearings.
76. Shearing and Bearing Values for High Tensile Structural Steel Shop Rivets, in accordance with B.S.S. No. 548/1934. Connected.
77. Permissible Compressive Stresses for High Tensile Structural Steel, manufactured in accordance with B.S.S. 548/1934.
78. Velocity of Flow in Pipes for a Given Delivery Connected.
79. Delivery of Water in Pipes for a Given Head
80. Working Loads in Mild Steel Pillar Shafts.
81. Involute Toothed Gearing Chart.
82. Steam Pipe Design. Chart showing Flow of Steam through Pipes.
83. Variation of Suction Lift and Temperature for Centrifugal Pumps.
84. Nomograph for Uniformly Distributed Loads on British Standard Beams.
85. " " " " "
86. " " " " " Connected.
87. Notes on Beam Design and on Use of Data Sheets, Nos. 84-5-6.
88. " " " " "
89. Curve Relating Natural Frequency and Deflection
90. Vibration Transmissibility Curve for Elastic Suspension Connected.
91. Instructions and Examples in the Use of Data Sheets, Nos. 89 and 90.
92. Pressure on Sides of Bunker.
93-4-5-6-7. Rolled Steel Sections.
98-99-100. Boiler Safety Valves.
101. Nomograph Chart for Working Stresses in Mild Steel Columns.

(Data Sheets are 2d to Members, 4d to others, post free.)

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